In this seminar we want to learn the theory of characteristic classes. Characteristic classes are invariants of vector bundles. They are among the most powerful tools in topology, for example for line bundles the characteristic classes completely determine the isomorphism class of the line bundle. There is also a close connection to the theory of (smooth) manifolds: for a closed manifold the characteristic classes of the tangent bundle are invariants of the manifold from which many properties of the manifold can be deduced. Characteristic classes even play an important role in homotopy theory because they are closely related to the generalized cohomology theory called $K$-theory which classifies vector bundles.

Those of you who attended the Topology I course this year already know an example of a characteristic class, namely the orientation character, which can be interpreted as a homomorphism

$$w_1 : H_1(M, \mathbb{Z}/2) \to \mathbb{Z}/2.$$  

This orientation character encodes the information of whether the manifold is oriented or not. It is the first Stiefel Whitney class. In the seminar we will introduce the higher Stiefel Whitney classes

$$w_n : H_n(M, \mathbb{Z}/2) \to \mathbb{Z}/2.$$  

We will however not do this in the language of homology, but in the language of cohomology (which can in this case be described by such homomorphisms).

In this seminar we will closely follow the classic book of Milnor and Stasheff [MS]. A required prerequisite to attend the seminar is knowledge of the topics covered in Topology I and Topology II (which can be attended in the same semester). More precisely we will need the language of homology, cohomology and manifolds. We try to repeat the important aspects quickly in the beginning. Everyone should look through chapter 1 of [MS] which repeats the notion of a smooth manifold before the seminar.

If you want to give a talk please write an e-mail with your favoured talk (or better a list of talks in case this is already gone) to nikolaus@math.uni-bonn.de I will try to update the list of talks regularly. If there are questions about the seminar also do not hesitate to write an E-mail. There will also be a short meeting Monday, February 2nd at 10:00 am in the Kleiner Hörsaal, Wegelerstraße 10. Due to time constraints (the lecture starts 10:15) we can only discuss some questions and then distribute the remaining topics. If you do not attend this meeting you have to come to my office at some other time that Monday (2.2.) to fill out the official applications sheet (including your Matrikelnummer and your signature). The time of the seminar will be Wednesday 16-18 in Room N0.008.
All talks should be planned such that there is enough time left for discussions, questions and additional remarks. Please submit a (handwritten) version of your talk 10 days before your talk.

Talk 1: Vectorbundles [MS][§2] ................................................................. Xianghui Zhong
Talk 2: Construction of vectorbundles [MS][§3] ........................................ Emma Brakkee
Talk 3: Stiefel-Whitney-classes I [MS][§4] ............................................. Tim Seynnaeve
Talk 4: Stiefel-Whitney-classes II [MS][§4] ............................................. Nicola Bellumat
Talk 5: Grassmann manifolds and universal bundles [MS][§5] .................. Lukas Kühne
Talk 6: A cell structure for Grassmann manifolds [MS][§6] ....................... Isabell Große-Brauckmann
Talk 7: Cohomology of the Grassmannian [MS][§7] ................................. Matthias Koch
Talk 8: Existence of Stiefel-Whitney classes [MS][§8] .............................. Michael Stahlhauer
Talk 9: Oriented bundles and the Euler class [MS][§9] .............................. Riccardo Pedrotti
Talk 10: The Thom isomorphism [MS][§10] ............................................. Luca Pol
Talk 11: Relation to smooth manifolds I [MS][§11] .................................... Davide Spriano
Talk 12: Relation to smooth manifolds II [MS][§11] ................................. Ester Cabezuelo Fernández
Talk 13: Chern classes [MS][§13 and §14] .............................................. Albert Schulz

The talks should cover the material contained in the respective chapters. But sometimes things can be shortened or left out if they were covered in the lecture (e.g. the notion of CW complexes in Talk 6). But you should of course read and understand all the details in the book. So please read and understand your chapter first and then we can discuss the exact details of the talks. I will also help you how to distribute the material for those talks that share a chapter (Talks 3,4 and Talks 11,12).

References