Robert Schrader
(1939–2015)

After a long, courageous fight against cancer, on November 29, 2015, *Robert Schrader* passed away.

Robert Schrader was born on September 12, 1939, in Berlin. In the aftermath of World War II his family moved to Norway, where Robert attended the primary school in Stavern and in Oslo. The life in Norway and the Scandinavian culture had a large, lasting impact on Robert. Robert's family returned to Germany in 1950, Robert entered the *Altes Realgymnasium* in Darmstadt, and afterwards he attended the *Herderschule* in Rendsburg, where he finished in 1959 with the Abitur.

He started his studies in physics in the summer of 1959 at the *University of Kiel*. There he passed the Vordiplom exam. Having studied one year at the *University of Zürich*, he went to Hamburg in 1962. Robert's thesis for the diploma degree, "Die Charaktere der inhomogenen Lorentzgruppe" [JS68] was supervised by H. Lehmann and H. Joos. He passed the exam for the diploma at the end of 1964. In the summer of 1965 he continued his studies at the *Seminar für Theoretische Physik an der Eidgenössischen Technischen Hochschule*, Zürich, and was appointed as assistant there from November 1967 on. Robert's advisor for his Ph. D. thesis, "On the Existence of a Local Hamiltonian in the Galilean Invariant Lee Model", was K. Hepp with R. Jost as co-advisor. He passed his doctoral exam in 1968, and the thesis was published in [S68]. He was awarded the "Habilitation" in theoretical physics by the *University of Hamburg* in 1971 with the paper "Das Yukawa Modell in zwei Raum-Zeit-Dimensionen" [S72].

In 1969 Robert followed an invitation of Arthur Jaffe to take a post-doc position as a research fellow at Harvard, and for a part of the time in the US he also worked at Princeton. A little while later Konrad Osterwalder came to Harvard as the second post-doc of Jaffe. At the suggestion of Jaffe, both Osterwalder and Schrader began to study the reconstruction theorem of Nelson, based on Symanzik's euclidean formulation of quantum field theory (QFT). They investigated the question of when a set of Schwinger functions, i.e. euclidean n–point functions of a field theory, determine the Wightman functions of the corresponding relativistic theory. The upshot, published in [OS73, OS75] [1], was a set of axioms, which contained *reflection positivity* (RP) — sometimes also called *Osterwalder–Schrader–positivity* — as a crucial property. In these articles they prove the *Osterwalder–Schrader–reconstruction–theorem* which states that one can reconstruct the Wightman functions of a relativistic QFT from the Schwinger functions of a euclidean theory satisfying the above mentioned set of axioms. In particular, RP is used for the construction of the physical Hilbert space together with the positive Hamiltonian of the theory. It is convenient to describe RP in terms of the fields: Consider a scalar classical or random field indexed by d–dimensional euclidean space-time, and define a reflection operator which maps the time coordinate into its negative. Then this operator acts in a natural way on functionals of the fields. Now consider an algebra of functionals of the fields (typically generated by products or exponentials of the fields), and consider the subalgebras which only involve the fields at positive (negative, resp.) times. Then RP is the property that every element in the positive (negative, resp.) subalgebra multiplied with its reflected, complex conjugated counterpart has a positive expectation value. As a consequence, this defines an inner product on the positive and negative subalgebras, which then is the starting point of the construction of a Hilbert space.

It was soon realized by Glimm, Jaffe and Spencer that by repeatedly using the Schwarz inequality which is implied by RP together with multireflections, one can establish estimates which lead to the proof of existence of phase transitions in certain quantum field theories. In the sequel the same idea was used by a large group of authors including Dyson, Fröhlich, Israel, Lieb, Simon, Spencer, among others, in the context of statistical mechanics. They turned RP in combination with the Peierls argument and infrared bounds into one of the fundamental tools for the treatment of phase transitions for models in statistical mechanics. In addition, reflection positivity was crucial in the study by K. Osterwalder and E. Seiler of the Wilson action for lattice gauge theory. Nowadays there is still very active research in structures exhibiting RP going on: For example, recently, Jaffe has studied RP in models with Majorana spins, and in models with topological order. A few weeks before he passed away, Robert published the manuscript [S15b], in which he proved RP for a model of simplicial gravity with a Hilbert action. So over the last forty years reflection positivity has gained importance as a fundamental concept in many scientific domains, also far beyond the one in which Osterwalder and Schrader discovered it.

In 1973 Robert accepted the offer of a professorship at the Department of

[1] A search with GOOGLE SCHOLAR shows that there are more than 900 citations of [OS73].

Physics at the *Freie Universität Berlin* (FU), and he stayed in this position until his retirement in 2005. In Berlin he set up the very ambitious program of a constructive approach to a ϕ^4–theory in four (euclidean) space–time dimensions. To this end, he gave a non-perturbative formulation of multiplicative renormalization within constructive QFT. The idea was to consider the three renormalization constants for the theory (mass counterterm, amplitude and vertex function renormalization) as functions of three parameters which are fixed as normalization constants (two involving the two point function, one involving the four point function), and to solve for the resulting equation as an implicit function problem. The cutoff theory is formulated on a periodic lattice, and in order to control the mapping from the normalization to the renormalization constants, Robert made use of various correlation inequalities, several of them being his own discoveries. However, in light of the results of Aizenman, Fröhlich and others on ϕ_4^4 at the beginning of the eighties, Robert's program remained uncompleted.

Another important thread of work of Robert, which began in the mid seventies, was in connection with Kato's inequality and the minimal coupling prescription of an external electromagnetic — or more generally Yang–Mills — field. The basic observation was that the Laplacian with minimal coupling can be controlled by the free Laplacian (i.e., without external field). On an informal level this becomes apparent if one considers the Feynman–Kac representation of its semigroup, in which the corresponding parallel transport operator enters as a unitary factor. Together with various authors Robert worked this out in a number of directions: a general, abstract version of Kato's inequality which implies the domination of a positive semigroup over another semigroup [HSU77] as conjectured by Simon, the application of this version of Kato's inequality to the semigroups generated by various Laplacians on Riemannian manifolds [HSU 80], quantum scattering theory, construction of a $P(\phi)_2$–theory in an external Yang–Mills field, semiclassical limits, inequalities for determinants, among others. The works [ST84] and [ST89] with Michael Taylor on Yang–Mills fields in the same line combine differential geometry with microlocal analysis, representation theory and ergodic dynamics to semiclassically estimate the partition function, Chern forms and eigenstates.

At the beginning of the eighties Robert began to work on Regge calculus and lattice gravitation. During a stay at the *IHES* in 1980 he met Werner Müller who was working on the lattice approximation of spectral invariants of Riemannian manifolds. They started a close collaboration which was then joined by Jeff Cheeger. After Robert and Werner Müller had returned to Berlin, they were separated by the Berlin Wall, since Müller lived and worked in East–Berlin. So Robert often went to the east side "as a tourist", sometimes together with Cheeger, and they were closely observed by the *Stasi*, the East–German secret service, while working in coffee shops.

Notwithstanding, this collaboration was born under the lucky star of mutual inspiration between quantum physics and differential geometry. In their first joint work [CMS84] they analyzed the simplicial approximation of the general (Lipschitz–Killing) curvatures. Whereas pointwise convergence for vanishing edge lengths turned out to be wrong, they could prove convergence in the sense of measures. In fact, Regge's calculus had been used by physicists for many years, taking convergence under subdivision for granted. The motivation for this work was to set up a euclidean path integral formulation of pure quantum gravity on (regularized) moduli spaces. This idea was taken up once again by Robert in [S15a] and [S15b]. The second focus of Schrader and Müller, this time with Borisov [BMS88], was the relation between a supersymmetric version of quantum scattering theory and index theorems. On open manifolds the heat kernel does not necessarily define a trace class operator, but the difference of the heat kernel of an asymptotically flat manifold and the heat kernel of the flat plane does. They calculated the supertrace in terms of topological invariants. In 1992 Bunke showed that this notion of a relative index coincides with the differential geometric notion of Gromov and Lawson of a relative index. After Müller's move to Bonn in the early nineties they continued the collaboration on various themes, in particular with Karowski on the invariants of 3–manifolds [KMS92], which were constructed by Turaev and Viro in terms of the 6–j symbols of quantum groups.

Around the time of the fall of the Berlin Wall, the mathematical physics group at the *FU* began to gain more and more recognition inside the German mathematical and theoretical physics communities. Together with Ferus, Pinkall and Seiler from the mathematics institute of *TU Berlin* and with Brüning (since 1995) and Friedrich from the mathematics institute of the *Humboldt Universität Berlin* Schrader initiated and managed the *Sonderforschungsbereich 288 — Differentialgeometrie und Quantenphysik*, a major collaborative research center funded by the *Deutsche Forschungsgemeinschaft.* It was two times prolonged and survived for three periods from 1992 until 2003. In addition he initiated with Brüning, Enß, Hirzebruch, Müller and Seiler a funding program for scientists from the former Soviet Union financed by the *Volkswagenstiftung*. Within one project of this program on classical integrable systems, which was headed by Novikov and Schrader, several scientists from the *Landau Institute* in Moscow repeatedly visited *FU*.

Another long standing collaboration in which many results and articles have been produced began in 1993, when Vadim Kostrykin visited Berlin within the framework of a Berlin–St. Petersburg exchange program. After a number of papers on (random) Schrödinger operators, ionization, cluster properties and related questions, partly in collaboration with others, they began to work on *metric graphs* as a domain in which one could try out established ideas in a new, non-trivial setting with important applications in many branches of science. Heuristically, a metric graph is a collection of finite or semi-infinite (i.e. isomorphic to the positive halfline) intervals which are connected at a number of the endpoints of the intervals. The standard metric on the intervals induces a metric on the resulting graphs. Metric graphs can be considered as idealized models for the configuration space of systems like quantum wires, nano-tubes, neural networks, and traffic models, to name just a few. Their work on metric graphs (in a later phase together with one of us (JP)) shows a large part of Robert's broad interests and knowledge. There are articles on: Kirchhoff–type rules on metric graphs, scattering theory and inverse scattering, applications to quantum wires and to information theory, the theory of walks on metric graphs with application to the analogue of Selberg's trace formula, the theory of (contraction, positivity preserving, Feller, heat) semigroups, an extensive theory of Laplace operators on metric graphs, the wave equation and in particular on its finite propagation speed, and the complete classification and construction of Brownian motions on metric graphs. Moreover, Robert had developed a theory of quantum fields on metrics graphs. Even though the underlying configuration space is locally just a one-dimensional linear space, interesting new aspects and problems are brought in by the boundary conditions at the vertices and by the non-trivial global topology of a given metric graph.

In our opinion — and this has also been stated by many others — it throws a somewhat peculiar light on the German university system that in spite of his truly outstanding academic achievements, Robert has never been appointed as a full professor. Also we want to mention that the decision of the Department of Physics at the *FU* to close mathematical physics with the retirement of Robert and of Michael Karowski in 2005 was to Robert's great disappointment.

Robert had a great passion for science, and in particular he was extremely curious about any subject in physics or mathematics. His knowledge in these two fields was impressively broad and deep, even to the point of very technical details. He was usually very enthusiastic about his projects, always full of new ideas, and worked with a high degree of energy. Robert was a charismatic teacher, and his lectures were always well-prepared, clear, and truly inspiring.

But Robert was generally curious about and interested in many other subjects, like music, history, philosophy, literature, and politics, to name just a few. In his youth he began to practice various kinds of sport, among them boxing, fencing, skiing and swimming, and the last two he continued to exert until late in his life. Robert had a fine sense for fairness, and he had told us that he derived it also from his sport activities, especially from fencing and boxing.

Robert was a person of integrity, very self-confident, and with a high sense of humor. He was often quite outspoken (which was not always welcome in some parts of the German academic community). Robert loved Norway, and spent almost every summer several weeks in the family's summer house in the south of Norway. All who knew him were deeply impressed by Robert's extraordinary courage and positive attitude, which he also maintained throughout his long illness.

Robert leaves behind Erika, his companion in life of the last decade, his wife Christa, their two children, Caroline and Stephan, his three sisters and his brother. For the scientific community his death is a tremendous loss.

We grieve for Robert Schrader, our mentor, colleague and friend,

Andreas Knauf, Jürgen Potthoff and Martin Schmidt

Acknowledgement: We heartily thank all colleagues, friends and family members of Robert who generously provided a large amount of information for this obituary.

Robert Schrader published over one hundred articles. Inevitably the selection of the ones mentioned in the obituary partly reflects our personal fields of interest.

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