

# Lecture Course: Advanced Global Analysis I (V4B3)

## The heat equation on Riemannian manifolds

The investigation of the heat equation on a Riemannian manifold is a powerful tool in spectral theory. It leads to deep relations between analysis, geometry and topology. One of its most outstanding applications is the heat equation proof of the Atiyah-Singer index theorem and its local version. Besides of this the heat equation has many other important applications in spectral theory.

The course is intended to be an introduction to the heat equation in the context of global analysis. We will begin with the study of its basic properties. One of the goals is the derivation of the asymptotic expansion of the heat kernel for an elliptic, positive definite differential operator on a compact Riemannian manifold. The coefficients of the asymptotic expansion provide a link between the spectrum of geometric elliptic operators on a Riemannian manifold and the geometry of the underlying manifold. Then we will discuss various applications of the heat kernel in spectral theory. If time permits we will also consider the main steps of the heat equation proof of the local index theorem.

**Prerequisites:** Global Analysis I + II.

**Date:** Tuesday 10 – 12, Thursday 8–19, room 1.008

### Literature

1. N. Berline, E. Getzler, M. Vergne, *Heat kernels and Dirac operators*, Springer-Verlag, Berlin, 2004.
2. P. Gilkey, *Invariance theory, the heat equation, and the Atiyah-Singer index theorem*. Studies in Advanced Mathematics. CRC Press, Boca Raton, FL, 1995.
3. I. Chavel, *Eigenvalues in Riemannian geometry*. Pure and Applied Mathematics, 115. Academic Press, Inc., Orlando, FL, 1984.
4. J. Jorgenson, S. Lang, *The ubiquitous heat kernel*. Mathematics unlimited – 2001 and beyond, 655–683, Springer, Berlin, 2001.
5. M. Atiyah, R. Bott, V.K. Patodi, *On the heat equation and the index theorem*. Invent. Math. **19** (1973), 279V330.
6. E.B. Davies, *Heat kernels and spectral theory*. Cambridge Tracts in Mathematics, 92. Cambridge University Press, Cambridge, 1990.