

Lecture Course: Advanced Global Analysis I (V4B4)

The Atiyah-Patodi-Singer index theorem

The index theorem of Atiyah-Patodi-Singer is an extension of the Atiyah-Singer index theorem to Dirac operators on manifolds with boundary. To obtain a Fredholm operator with a well defined index one has to introduce boundary conditions. Appropriate boundary conditions for Dirac operators were introduced by Atiyah, Patodi and Singer in [APS1]. These are the so called APS boundary conditions. In the index formula appears a new term which is attached to the boundary. This is the so called η -invariant which is defined in terms of the spectrum of the induced Dirac operator on the boundary. The η -invariant by itself has many interesting applications in geometry and topology. The derivation of the index theorem and its applications will be discussed in detail in the course. A different approach to the APS index problem has been developed by Melrose [Me]. The compact manifold with boundary is replaced by a manifold with a cylindrical end. In this set up, the index is defined in the L^2 -sense, which means that we consider the space of square integrable sections in the kernel of the Dirac operator. To derive an index formula for the L^2 -index, one has to develop spectral theory on manifolds with cylindrical ends. The new feature is the presence of a non-empty continuous spectrum for the Dirac operator. Analysis on non-compact manifolds is an important direction of current research in global analysis. In the course I will develop some of the basic methods in the context of manifolds with cylindrical ends.

Literatur

- [APS1] Atiyah, M. F.; Patodi, V. K.; Singer, I. M. Spectral asymmetry and Riemannian geometry. I. Math. Proc. Cambridge Philos. Soc. 77 (1975), 43–69.
- [APS2] Atiyah, M. F.; Patodi, V. K.; Singer, I. M. Spectral asymmetry and Riemannian geometry. II. Math. Proc. Cambridge Philos. Soc. 78 (1975), no. 3, 405–432.
- [APS3] Atiyah, M. F.; Patodi, V. K.; Singer, I. M. Spectral asymmetry and Riemannian geometry. III. Math. Proc. Cambridge Philos. Soc. 79 (1976), no. 1, 71–99.
- [Me] Melrose, Richard B.; The Atiyah-Patodi-Singer index theorem. Research Notes in Mathematics, 4. A K Peters, Ltd., Wellesley, MA, 1993.