Lecture Course: Advanced Global Analysis II (V4B4)

Spectral theory of automorphic forms II

In the simplest case, spectral theory of automorphic forms means the study of the spectrum of the Laplace operator on hyperbolic surfaces of finite area. Such a surface is of the form $\Gamma\backslash\mathbb{H}$, where $\mathbb{H}$ is the upper half-plane, equipped with the hyperbolic metric, and $\Gamma \subset \mathrm{SL}(2,\mathbb{R})$ is a discrete subgroup of finite covolume, which acts on $\mathbb{H}$ by fractional linear transformations. In general, it means harmonic analysis on a locally symmetric space $\Gamma\backslash G/K$ of finite volume.

The study of the spectrum of the Laplace operator on such manifolds (or orbifolds) has many important links to other fields in mathematics such as number theory, representation theory of reductive groups over local and global fields, differential geometry, and mathematical physics.

The main result of part I, which I tought in the last semester, is the description of the spectral resolution of the Laplace operator, acting in the Hilbert space of square integrable functions on a hyperbolic surface. At the beginning I will recall this result without proof.

In present course I will continue to study the spectrum of the Laplace operator on hyperbolic surfaces. One of the main goals is the derivation of the Selberg trace formula, which is one of the most powerful analytic tools in the theory of automorphic forms. It will be applied to the study of the discrete spectrum of the Laplace operator, which is one of the main issues in the theory of automorphic forms. If time permits, I will discuss the relation with representation theory, the adelic point of view, automorphic L-functions, and some aspects of arithmetic quantum chaos.

References

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