Lecture Course: Topics in Global Analysis II (V4B4)

The Atiyah-Singer index theorem: The heat equation proof

The Atiyah-Singer index theorem computes the index of an elliptic differential operator on a compact manifold in terms of topological invariants of the manifold and the differential operator. It generalizes results like the Gauss-Bonnet-Chern theorem, the Riemann-Roch-Hirzebruch thoerem and the Hirzebruch signature theoerem. The index theorem has many important applications in topology, geometry, mathematical physics, etc. In the course I will use the heat equation method to establish the local version of the index theorem for Dirac type operators. If time permits, I will also discuss the Atiyah-Patodi-Singer index theorem for manifolds with boundary. This theorem is based on the local index theorem on closed manifolds.

Literatur

[1] N. Berline, E. Getzler, M. Vergne, *Heat kernels and Dirac operators.* Grundlehren, Springer-Verlag, Berlin, 2004.

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[5] M.F. Atiyah, V.K. Patodi, I.M. Singer, *Spectral asymmetry and Riemannian geometry*, Math. Proc. Cambridge Phil. Soc. **77** (1975), 43 – 69