Prof. Dr. W. Müller
Dr. J. Matz

Sommersemester 2014

Seminar on Global Analysis S4B3: Spectral theory and Riemannian Geometry

The seminar is concerned with the relationships between the geometric structures of Riemannian manifolds and spectra of canonically defined differential operators on this manifold. The case of the Laplace operator on a closed Riemannian manifold has been most intensively studied. Problems studied in this field are devided into direct and inverse problems.
Inverse problems seek to identify features of the geometry from information about the eigenvalues of the Laplacian. One of the earliest results of this kind is Weyl's law – due to Hermann Weyl – which shows that the volume of a bounded domain in Euclidean space is determined by the asymptotic behavior of the eigenvalues for the Dirichlet boundary value problem of the Laplace operator. This question is usually expressed as "Can one hear the shape of a drum?", a popular phrase due to Mark Kac. A refinement of Weyl's asymptotic formula obtained by Pleijel and Minakshisundaram produces a series of local spectral invariants involving covariant differentiations of the curvature tensor, which can be used to establish spectral rigidity for a special class of manifolds. However, in general, the spectrum of the Laplace operator does not determine the underlying manifold up to isometry. This is called the problem of isospectrality.
Direct problems attempt to infer the behavior of the eigenvalues of a Riemannian manifold from knowledge of the geometry. The solutions to direct problems are typified by the Cheeger inequality which gives a relation between the first positive eigenvalue and an isoperimetric constant (the Cheeger constant). Many versions of the inequality have been established since Cheeger's work.
In the seminar we will discuss some of these aspects.

Prerequisites: Geometry I, Global Analysis I

Date: Tuesday, 16:15 – 17:45, Room 0.008

Distribution of talks: Thursday, February 6, 14:00, Room N 0.008, or by e-mail

Literature:

1. I. Chavel, Eigenvalues in Riemannian Geometry, Academic Press, 1984.
2. M. Reed, B. Simon, Methods of modern mathematical physics I – IV, Academic Press, 1975.
3. H. Triebel, Higher Analysis, Leipzig 1992.
4. M. Berger; P. Gauduchon; E. Mazet, Le spectre d'une variété riemannienne. LNM Vol. 194 Springer-Verlag, Berlin-New York 1971

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Talks

1. The spectral theorem for selfadjoint operators I
2. The spectral theorem for selfadjoint operators II
3. Absence of singular continuous spectrum
4. Mini-max principle for eigenvalues
5. Cheeger inequality
6. Weyl law
7. Isospectrality I
8. Isospectrality II
9. Seperation of variables and applications
10. Harmonic polynomials and spectrum of the sphere
11. Harmonic oscillator
12. Hydrogen atom