

Prof. Dr. W. Müller
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Sommersemester 2012
Seminar on Global Analysis S4B3:
The Atiyah-Singer index theorem

The Atiyah-Singer index theorem computes the index of an elliptic differential operator on a compact manifold in terms of topological invariants of the manifold and the differential operator. The index theorem is a far reaching generalization of classical results like the Gauss-Bonnet theorem, the Riemann-Roch-Hirzebruch theorem, the signature theorem of Hirzebruch and the Lefschetz fixed point theorem. It has many applications in topology, geometry, mathematical physics, etc. In the seminar we will deal with the cohomological version of the index theorem. This requires a number of topological foundations such as K-theorie, characteristic classes, and the theory of genera in the sense of Hirzebruch.

Prerequisites: Global Analysis I + II, Geometrie I.

Date: Thursday, 14:15, room 0.008

Distribution of talks: Thursday, February 2, 14:15, room 0.008, or by e-mail

Literature:

1. M.B. Lawson u. M.-L. Michelson: Spin Geometry, Princeton Univ. Press, 1989.
2. R. Palais: Seminar on the Atiyah-Singer index theorem, Annals of Math. Studies **57**, Princeton univ. Press.
3. F. Hirzebruch: Topological methods in algebraic geometry, Springer, 1995
4. D. Husemoller: Fibre bundles, Graduate Texts in Mathematics, 20. Springer-Verlag, New York, 1994.
5. M.F. Atiyah: K-theory, Addison-Wesley Publishing Company, Advanced Book Program, Redwood City, CA, 1989.
6. M.F. Atiyah, I. M. Singer: The index of elliptic operators. I. Ann. of Math. (2) **87** 1968 484–530.

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Talks

I. K-theory

1. The K-functor and its properties, exact sequence, etc.
2. The Bott periodicity theorem.
3. The Thom isomorphism.
4. Elliptic operators and K-theory.

II. Characteristic classes

5. Chern classes of a complex vector bundle.
6. Pontrjagin classes of a real vector bundle.
7. The Chern character and Thom isomorphism.
8. Multiplicative sequences and genera.

III. The topological and analytic index

9. The topological index and its properties.
10. The analytic index and K-theory.
11. The cohomological version of the index theorem.
12. Examples of the index theorem.