

S4B3 Graduate Seminar Global Analysis WS 2017/2018

Spectral theory of automorphic forms

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A preliminary meeting will be held between 25-09-2017 and 6-10-2017

Interested students can contact us by email

Content

Let $X = G/K$ be a Riemannian symmetric space and Γ a discrete torsion free group of isometries of X . An automorphic form for Γ is an eigenfunction of the Laplacian on $\Gamma \backslash X$. The goal of the seminar is to describe, for a Fuchsian group acting on the hyperbolic plane, the relation between the spectrum of the Laplacian and the Selberg trace formula associated to a discrete subgroup $\Gamma \subset PSL_2(\mathbf{R})$. We will touch upon connections with number theory as well.

Prerequisites

(Riemannian) manifolds, Lie groups, complex analysis, self-adjoint operators in Hilbert space.

Recommended literature

Our guiding text will be *Intrduction to the Spectral Theory of Automorphic Forms* by H. Iwaniec. In addition we the following comprehensive sources are useful:

1. A. Deitmar, *Automorphic forms*
2. S. Gelbert, *Automorphic forms on adèle groups*
3. I.M. Gelfand, S. Graev and I.I. Piatetskii-Shapiro, *Representation theory and automorphic functions*
4. D.A. Hejhal, *The Selberg trace formula I & II*
5. W. Müller, *Spectral theory of automorphic forms*, lecture notes
6. A. Selberg, *Harmonic analysis in Collected papers I*

Tentative breakdown of talks

Generally, each talk covers a chapter of Iwaniec's book. Content and order may be adapted in consultation with the participants.

1. Review of the hyperbolic plane (Iwaniec Ch.1);
2. Fuchsian groups (Iwaniec Ch.1, Ch. 2).
3. Automorphic forms, modular forms for $SL(2, \mathbb{Z})$ (Iwaniec Ch.3, Deitmar Ch.2).
4. The discrete spectrum of the Laplace operator (Iwaniec Ch. 4, Gelfand Ch. 1.6).
5. The automorphic Green function (Iwaniec Ch.1, Ch.5).
6. Eisenstein series (Iwaniec Ch.6).
7. The continuous spectrum of the Laplace operator (Iwaniec Ch.7, Gelfand Ch. 1.6)
8. Fourier coefficients of Maass forms (Iwaniec Ch.8).
9. Kloosterman Sums (Iwaniec Ch.9).
10. The trace formula (Iwaniec Ch.10, Gelfand Ch.1.5).
11. The Selberg zeta function (Iwaniec Ch.10).
12. Weyl's law (Iwaniec Ch.11).