Research statement

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Fields: Forcing, Descriptive Set Theory, Set Theory of the Reals.

In *Set Theory of the Reals*, one studies the connection between certain properties of sets of reals like *regularity properties* (e.g. Baire property, Lebesgue measurability) and the *complexity* of sets. For example, every analytic set has the Baire property and is Lebesgue measurable. But when we replace "analytic" with a higher complexity level like $\mathbf{\Sigma}^1_2$ or $\mathbf{\Delta}^1_2$, the issue becomes meta-mathematical and the theory of forcing comes in. A typical such example is Solovay's characterization theorem [Sol69] which states:

$$\text{“Every } \mathbf{\Sigma}^1_2 \text{ set is Lebesgue measurable} \iff \forall x \text{ (the set of random reals over } \mathbf{L}[x] \text{ is null)”}$$

We can generalize this result as follows: if $\mathbb{P}$ denotes an arbitrary forcing partial order, we associate to it an *algebra of measurability* $\mathcal{A}(\mathbb{P})$ as well as a *null-ideal* $\mathcal{I}(\mathbb{P})$. A Solovay-style theorem then reads as follows:

$$\text{“Every } \mathbf{\Sigma}^1_2 \text{ set is in } \mathcal{A}(\mathbb{P}) \iff \forall x \text{ (the set of } \mathbb{P}\text{-generic reals over } \mathbf{L}[x] \text{ is in } \mathcal{I}(\mathbb{P}))\text{”}$$

Other variants of such results, also called *Judah-Shelah-style theorems*, are

$$\text{“Every } \mathbf{\Delta}^1_2 \text{ set is in } \mathcal{A}(\mathbb{P}) \iff \text{for every } x\text{, there is a } \mathbb{P}\text{-generic real over } \mathbf{L}[x]\text{”}$$

Such theorems have been proved for different $\mathbb{P}$, among others in [JuShe89, BrLö99, BrHaLö05].

The aim of my research is to generalize these theorems in such a way as to cover a wide variety of forcing notions. One direct apporach is finding conditions on $\mathbb{P}$, so that if those are satisfied, then a Solovay-style or a Judah-Shelah-style theorem can be proved for $\mathbb{P}$. Another approach is via *cardinal invariants*: for example, if $\mathsf{add}(\mathbb{P}) \leq \mathsf{add}(\mathbb{Q})$ then this gives a good indication that a Solovay-style theorem for $\mathbb{P}$ implies one for $\mathbb{Q}$. The goal of my project is to make these "indications" precise and find out more about the underlying theory.

A starting point for my research is Zapletal's analysis of a similar problem in [Z04], as well as some ideas introduced in [BrHaLö05]. Also, this project may potentially be carried out in partial collaboration with my fellow PhD student Daisuke Ikegami, who has already obtained new results in this direction [Ik06].

References

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