Research Statement*

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My primary research interest is **descriptive set theory**. In my PhD thesis I developed a new concept in the theory of **Hurewicz tests**. Roughly speaking, a Hurewicz test allows to witness the class of sets in the Borel hierarchy. My main revelation concerns the Hurewicz tests for the Borel class Σ^0_ξ $(0 < \xi < \omega_1)$, as follows. For every fixed ordinal $\xi < \omega_1$ it is possible to endow the Polish space 2^ω with a finer Polish topology in such a way that every Σ^0_ξ is either "almost empty" or of second category in this finer topology. This makes possible the application of the Baire Category Theorem for Σ^0_ξ sets. Accordingly, this concept turned out to have **numerous Baire Category Theorem-like applications**. In particular,

- it implies that if the union of less than $\mathrm{cov}(\mathcal{M})$ many Σ^0_ξ sets is Borel then it is Σ^0_ξ (see [6]); this reproves a theorem of J. Stern.

- it allows the construction of Hurewicz test sets for generalized separation of analytic sets (see [3]).

- under the assumption of the continuum hypothesis, it refutes a question of A. Miller concerning analytic ideals (see [7]); the question had previously been refuted in an unpublished work of A. Kechris and M. Zelený using $V = L$;

- using these test pairs it is possible to show that there is no monotone presentation for Borel sets (see [5]); this result is a natural counterpart of some results of M. Elekes and A. Máthé, and of A. Andretta, G. Hjorth and I. Neemann.

Currently I am interested in the study of **σ-ideals of compact** sets and of **Borel equivalence relations**. I managed to answer in the negative a question of A. Kechris by constructing a G_δ σ-ideal of compact subsets of 2^ω which contains all the singletons but does not contain all the compact subsets of any dense G_δ set in 2^ω (see [4]). This result indicates that G_δ σ-ideals can exhibit wilder behavior than expected. Concerning Borel equivalence relations, currently I study relations E satisfying $l^1 \leq_B E \leq_B l^\infty$. In our joint work with M. Vizer we obtained that, contrary to present-day belief, there are many other Borel equivalence relations E satisfying $l^1 \leq_B E \leq_B l^\infty$ than just the l^p ones or the direct sums of these (see [8]).

As a member of the set theory and general topology workgroup of the Rényi Institute, led by I. Juhász, I am active in **set theoretic topology**, as well. During a research stay in the workgroup of D. Preiss at University College London I worked on l-equivalence of topological spaces (see [2]). In my recent joint work with M. Elekes and L. Soukup we examined the problem whether for some cardinals κ and λ, in a given topological space, a κ-fold cover by sets with various geometric and topological properties can be split into λ many disjoint subcovers (see [1]).

References

[1] M. Elekes, T. Mátrai, L. Soukup, On the splittability of infinite covers, preprint.

[2] T. Mátrai, A characterization of spaces l-equivalent to the unit interval, *Topology Appl.* 138 (2004), no. 1-3, 299–314.

[3] T. Mátrai,Hurewicz test sets for generalized separation and reduction, *Math. Proc. Cambridge Phil. Soc.*, to appear.

[4] T. Mátrai, Kenilworth, preprint.

[5] T. Mátrai, On monotone presentations of Borel sets, submitted.

[6] T. Mátrai, On the closure of Baire classes under transfinite convergences, *Fundamenta Mathematicae*, 183 (2004), 157–168.

[7] T. Mátrai, Π^0_2-generated ideals are unwitnessable, submitted.

[8] T. Mátrai, M. Vizer, On Borel equivalence relations between l^1 and l^∞, joint paper with M. Vizer, preprint.

*The publications on which this research statement is based can be found at http://www.renyi.hu/∼matrait