Research statement

Piotr Borodulin–Nadzieja
pborod@math.uni.wroc.pl
http://www.math.uni.wroc.pl/~pborod
Mathematical Institute,
University of Wroclaw, Poland

I'm interested in applications of set theory to measure theory, topology and functional analysis. Below I list my current areas of interest.

Measure games. In [7] Fremlin introduced infinite games defined in the same way as Choquet (Banach–Mazur) games in topology, but in which players use elements of a σ–algebra of measurable sets instead of open sets. Among other results Fremlin showed that if we play Borel subsets of a Polish space, then the second player has a winning strategy. In [1] Grzegorz Plebanek and I presented an easier and more intuitive proof of this fact. One of the interesting open problems in this field is: is there a measure space such that the second player has a winning strategy but has no winning tactic? Debs in [4] showed an example of topological space with this property.

Counterexamples via Stone isomorphism. There is a method of constructing peculiar topological (or Banach) spaces using Boolean algebras. The idea is to encode some combinatorial properties in a Boolean algebra (often, a subalgebra of $P(\omega)$) to obtain an interesting topological space as a Stone space of this algebra. Longstanding Efimov problem is a good example of a problem, which could be solved by this method. Efimov asked if there is an infinite compact space without nontrivial convergent sequences and without a copy of $\beta\omega$ (a *Efimov space*). There are partial (negative) answers to this problem, e.g. under CH or a certain assumption on the splitting number $\mathfrak{s}$. In [2] I give some constructions (in ZFC and under MA) of spaces satisfying certain Efimov–like properties. In [3] Grzegorz Plebanek and I showed that the existence of Banach space which has a Mazur Property and does not have a Gelfand-Phillips property (properties connected to the theory of Pettis integrals, which have something to do with convergence of sequences of points and of measures) is equivalent to certain cardinal invariants inequality. One of the tools used in constructions of peculiar compact spaces via Stone isomorphisms is *minimally generated Boolean algebra* (see [8]). They were used in [5], [9] for constructions of Efimov spaces. I investigated (mainly measure theoretic) properties of minimally generated Boolean algebras in [2].

Separable measures. There is a well–known question (MRP(separable) in terms of [6]) about a characterization (combinatorial, topological) of the class of Boolean algebras admitting only separable measures. In [2] I showed that every Boolean algebra either admits a uniformly regular measure or it carries a measure which is non–separable and that the class of minimally generated Boolean algebras is a (quite rich) class of Boolean algebras which carry only separable measures.

The combinatorial structure of $P(\omega)$. Seeking interesting examples of topological spaces via Stone isomorphism we usually need to solve certain combinatorial problems concerning the structure of $P(\omega)$, in particular we have to deal with inequalities on cardinal invariants (such as $\mathfrak{p}$, $\mathfrak{b}$, $\mathfrak{h}$). In my work the cardinal invariants connected to the ideal of asymptotic density zero sets are particularly important (see [3]).

References

[1] P. Borodulin–Nadzieja, G. Plebanek, *On compactness of measures on Polish spaces*, Illinois J. Math. **49** (2005), 531–545.

[2] P. Borodulin–Nadzieja, *On measures on minimally generated Boolean algebras*, Topology Appl. **154** (2007) 3107–3124.

[3] P. Borodulin–Nadzieja, G. Plebanek *Mazur property, Gelfand–Phillips property and special compactifications of ω*, preprint available via email.

[4] G. Debs, *Strategies gagnantes dans certains jeux topologiques*, Fund. Math. **126** (1985), 93–105.

[5] A. Dow, *Efimov spaces and the splitting number*, Topology Proc. **29** (2005), 105–113.

[6] , M. Dzamonja, *Measure Recognition Problem*, Philosophical Transaction of the Royal Society **364**, 2006.

[7] D.H. Fremlin, *Weakly α–favourable measure spaces*, Fund. Math. **165** (2000), 67–94.

[8] S. Koppelberg , *Minimally generated Boolean algebras*, Order **5** (1989), 393–406.

[9] S. Koppelberg, *Counterexamples in minimally generated Boolean algebras*, Acta Univ. Carolin. **29** (1988), 27–36.