RESEARCH STATEMENT

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My research area is in Generic Absoluteness, more specifically on Projective Absoluteness and its relations with Descriptive Set Theory. My advisor is Joan Bagaria from the University of Barcelona and ICREA.

Projective Absoluteness studies the invariance of the truth values of the statements about real numbers between a model and its generic extensions.

Formally, for a model M of ZFC^*, $n \in \omega$, and a poset $\mathbb{P}$, we say that M is $\mathbf{\Sigma^1_n}$-$\mathbb{P}$-*absolute* if $M \prec_{\Sigma^1_n} M^{\mathbb{P}}$, i.e., the truth value of every Σ^1_n-formula is invariant by forcing with $\mathbb{P}$, that is, for any Σ^1_n-formula $\varphi(x)$, and for every real $a \in M$, $M \vDash \varphi(a)$ iff $M^{\mathbb{P}} \vDash \varphi(a)$.

Shoenfield [6] proved in ZFC that the truth value of every Π^1_2-statement (the negation of a Σ^1_2-statement) with real parameters is invariant under forcing, that is, $\mathbf{\Pi^1_2}$-$\mathbb{P}$-absoluteness holds for every poset $\mathbb{P}$. Since the statement which says "*there is a nonconstructible real*" can be formalized by a Σ^1_3 formula and it is false in L but can be forced to be true, Σ^1_3-absoluteness cannot be proved in ZFC. Indeed, the consistency strength of $\mathbf{\Sigma^1_3}$-absoluteness is a reflecting cardinal [2, 3].

Surprisingly, Projective Absoluteness is strongly connected with the topological regularity properties of projective sets of reals, those sets definable by some projective formula (Σ^1_n or Π^1_n) by means of real parameters. These are properties that assert that the sets resemble in some topological aspect very much to a Borel set and, in this sense, that they have a nice behaviour.

By results of Bagaria [1], Judah and Shelah [5], and Ikegami [4], we know that for some natural forcing notions $\mathbb{P}$, there is a regularity property $P_{\mathbb{P}}$ of sets of reals such that the following are equivalent:

(1) $\mathbf{\Sigma^1_3}$-$\mathbb{P}$-absoluteness,
(2) Every $\mathbf{\Delta^1_2}$ set of reals has the property $P_{\mathbb{P}}$,

where a set of reals is $\mathbf{\Delta^1_n}$ if it can be defined at the same time by a Σ^1_n-formula and by a Π^1_n-formula by means of real parameters.

On the other side, Feng, Magidor, and Woodin proved in [3] that the following are also equivalent:

(1) $\mathbf{\Sigma^1_3}$-$\mathbb{P}$-absoluteness for every set forcing $\mathbb{P}$,
(2) Every $\mathbf{\Delta^1_2}$ set of reals is universally Baire,

where the universal Baireness is a topological property which implies all the classical regularity properties. A set is *universally Baire* if for every infinite cardinal κ, for every continuous function $f\colon \kappa^\omega \to \omega^\omega$, $f^{-1}[A]$ has the Baire property in κ^ω. Universal Baireness allows one to describe a set with this property using trees whose projections are complementing each other in any generic extension.

My research goal now is to study if there is some equivalence of this kind for every forcing $\mathbb{P}$, i.e., isolate for every forcing $\mathbb{P}$ (or for some specific classes of forcings such as ccc, proper, or semiproper, for example) some topological property inspired in universally Baireness which could give this kind of equivalence.

References

[1] J.Bagaria, *Definable forcing and regularity properties of projective sets of reals*, Ph. D. Thesis. University of California, Berkeley, 1991.

[2] J. Bagaria, Sy D.Friedman, *Generic absoluteness*, Ann. Pure Appl. Logic 108 (2001), no. 1-3, 3–13.

[3] Q. Feng, M. Magidor, and H. Woodin, *Universally Baire sets of reals*, in Set theory of the continuum, MSRI Publications 26, pp. 203–242, 1992.

[4] D. Ikegami, *Projective Absoluteness under Sacks forcing*, Master Thesis.

[5] H. Judah and S. Shelah, $\mathbf{\Delta^1_2}$ *sets of reals*, Ann. Pure Appl. Logic **42** (1989), no.3, 207–223.

[6] J.R. Shoenfield, *The problem of predicativity,* Essays on the foundations of mathematics, Magnes Press, Hebrew Univ., Jerusalem, 1961, pp. 132–139.