RESEARCH STATEMENT

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1. Singular cardinal combinatorics in the context of large cardinals

The driving interest of my researches has been the study of singular cardinal combinatorics in models of strong forcing axioms like MM or PFA. There are several problems in this area of which we get a clear picture assuming strong forcing axioms. In particular the techniques that led me to a proof of SCH from PFA are useful to study many other related issues:

- **Cardinal arithmetic.** The singular cardinal hypothesis SCH holds in models of MM or PFA and there are currently a number of proofs of SCH from MM or from PFA (for a proof of SCH from PFA see [3]).
- **The approachability ideal.** On one hand MM implies that there is a stationary subset of $S^{\aleph_{\omega+1}}_{\aleph_1}$ in the approachability ideal on $\aleph_{\omega+1}$. On the other hand, under MM, $\aleph_1$ is the unique cofinality for which the approachability ideal does not contain a relative club [2].
- **Saturation properties of models of strong forcing axioms.** We have the following results [3]:
  - If V is model of MM and W is an inner model with the same cardinals then $\mathtt{cof}(\kappa)^W > \aleph_1$ if and only if $\mathtt{cof}(\kappa) > \aleph_1$ for all cardinals κ.
  - if V models MM and is a set forcing extension of W and V and W have the same cardinals, then $[Ord]^{\aleph_1} \subseteq W$.

The above results suggest that the following should hold:

Conjecture 1.1. *Assume* MM *and let W be an inner model with the same cardinals. Then:*

(1) $[Ord]^{\aleph_1} \subseteq W$,
(2) *κ is regular iff κ is regular in W.*

A positive answer to these questions would suggest that a model of MM is essentially characterized by its cardinal structure, since any submodel which computes correctly the cardinals resembles closely to the universe.

A suitable version of the above results and conjectures can be stated also for supercompact cardinals.

I would like to continue to investigate the ground for the above conjectures and also to attack the problem of the eventual consistency of $\aleph_\omega$ being a Jónsson cardinal. My first step would be to try to prove that $\aleph_\omega$ is not Jónsson in a model of MM. König [1] has already shown that MM is consistent with $\aleph_\omega$ not being Jónsson.

References

[1] B. König. Forcing indestructibility of set theoretic axioms. *Journal of Symbolic Logic*, 72:349–360, 2007.

[2] A. Sharon and M. Viale. Reflection and approachability. in preparation.
[3] M. Viale. A family of covering properties. to appear in Mathematical Research Letters, 18 pages.