SOME APPLICATIONS OF REFLECTION PRINCIPLES TO PCF THEORY

MATTEO VIALE

We present some application of reflection principles to the analysis of the partial order of reduced product of regular cardinal. The guiding example being the study of the partial order $(\prod_n \aleph_n, <^*)$, where $f <^* g$ if $f(n) \geq g(n)$ for finitely many n. The main result is that a reflection principle on $\aleph_2$ which is equiconsistent with $\aleph_2$ being weakly compact in L and which follows from Martin's maximum implies that club many points of cofinality $\aleph_2$ below $\aleph_{\omega+1}$ are approachable. This is obtained by a combination of two theorems: one by me and the other by Assaf Sharon. We then link these results to the study of the transfer principles

$$(\aleph_{\omega+1}, \aleph_\omega) \to (\aleph_{n+1}, \aleph_n).$$

In particular results of Shelah show that this Chang conjecture fails if $n > 2$. Under the assumption of our main theorem we show that it fails also for $n = 1, 2$. Levinsky Magidor and Shelah on the other hand have shown that

$$(\aleph_{\omega+1}, \aleph_\omega) \twoheadrightarrow (\aleph_1, \aleph_0)$$

is consistent.

Kurt Gödel Research Center for Mathematical Logic, Vienna
E-mail address: `matvial at logique.jussieu.fr`