RESEARCH STATEMENT

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I am interested in the interplay between set theory and algebra and its applications to the theory of modules over commutative rings. In particular my research focuses on structure theorems and realization theorems for Abelian Groups - a category that is usually a good test-class for possible results on larger modules classes.

(Abelian) Groups are defined by a set of elements and their defining relations. For instance, the well-known divisible Prüfer group $\mathbb{Z}(p^\infty)$ for some prime p is generated by elements $\{x_0, x_1, x_2, \cdots\}$ such that $px_0 = 0, px_1 = x_0, px_2 = x_1$. More generally, Crawley and Hales call a group simply presented if its defining relations are of the form $nx = y$ or $nx = 0$ where n is a positive integer. Obviously, there are much more complicated groups increasing the complexity of the relations. A group homomorphism is a map that respects those relations and very often homomorphisms and automorphisms of groups give you much information about the group itself. Thus the idea is to use combinatorial principles like the Black Boxes, properties of cardinals and a set-theoretic machinery to construct abelian groups (or modules over more general rings) having many, only a few or prescribed homomorphisms.

As an example transitivity properties of modules are subject of my studies. Transitivity, weak transitivity and full transitivity provide examples of modules with a rich structure and the property that every two elements can be mapped onto each other under certain natural and necessary assumptions. The notion of transitivity and full transitivity goes back to I. Kaplansky while weak transitivity is a new concept and is of a categorical nature. Similarly I am interested in the structure of the group of extensions Ext(G,H) for R-modules G and H. Since the solution of the famous Whitehead problem by Saharon Shelah it is well-known that this structure, in particular the vanishing of Ext(G,Z), depends on the underlying set theory. In some models a characterization of Ext(G,Z) is known - in others its understanding is beyond reach. Again, infinite combinatorics and set theory can be used to prove structure theorems like singular compactness theorems, the existence of universal modules etc. This has also applications in tilting and cotilting theory.

Since most of the constructions and techniques I am using involve (infinite) combinatorics, geometric objects and set-theoretic as well as model-theoretic arguments I am also very much interested in these areas, e.g. cardinal arithmetic and axiomatic set theory.

For my publications and further details on my research interests please see my webpage or contact me directly by e-mail: lutz@math.uni-bonn.de