Research Statement (Gunter Fuchs)

Inner Model Theory

I compared the fine structures that arise in two different approaches to constructing extender models (definable submodels of a universe of set theory which have a canoncial structure while reflecting strong axioms of infinity true in the universe), as pursued by William Mitchell and John Steel on the one hand, and Sy Friedman and Ronald Jensen on the other. I established a one-to-one correspondence between the two models' building blocks. I developed a method of translating formulae between the corresponding structures. It also translates iteration strategies for these structures.

In my post-doctoral work, I pursued the problem of iterability, i.e., of the existence of iteration strategies, further, as it is central to inner model theory.

Forcing

Příkrý forcing: I introduced a generalization and characterized the corresponding generic sequences combinatorially, connecting them to inner model theory both ways.

Forcing axioms: The maximality principle for $<\kappa$-closed forcings, for a regular cardinal κ, says that any statement that can be forced by a $<\kappa$-closed forcing to be true in such a way that it stays true in any further forcing extension of that kind, is already true. I investigated these principles and will look at their connections to modal logic. They have many interesting consequences, and can be combined. But the strongest thinkable combinations of these principles is inconsistent. Can they hold at every regular cardinal below some cardinal satisfying some strong axiom of infinity? This is a topic that I am currently investigating.

Set Theoretic Geology: Together with Joel Hamkins and Jonas Reitz. The starting point is the result of Richard Laver, which was obtained independently by Hugh Woodin, that the ground model is definable in each of its forcing extensions. We are now trying to define and investigate canonical inner models of a given model, using this method of "inverting forcing".

Interactions between Algebra and Set Theory

The automorphism tower of a centerless group: A centerless group embeds into its automorphism group, which is again a centerless group. Iterating this process leads to a sequence of centerless groups, each embedded into the next. At limits, it is possible to form the direct limit; one gets yet another centerless group, and so the process can be continued transfinitely. The groups occurring in this sequence form the automorphism tower of the original group. The first index of a group in the automorphism tower that is isomorphic to the next group is called the height of the automorphism tower. In work on rigidity degrees of Souslin trees, Joel Hamkins and I developed methods for subtly coding automorphisms of one tree into branches of another tree. Building on this, we were able to prove that in L, there are groups whose automorphism tower heights are highly malleable by forcing: The height of the automorphism tower of the very same group can be very different in different forcing extensions of L.

Large Cardinals

Combined closed maximality principles up to a large cardinal: I use "Woodinized" versions of known large cardinal concepts - these are ideas due to Matthew Foreman, and techniques of lifting elementary embeddings to generic extensions, mostly of the kind introduced by Silver.

Indestructible Weak Compactness: In the aforementioned area I came across the concept of an indestructible weakly compact cardinal quite naturally: If the closed maximality principle holds on a measure one set below a measurable cardinal, then there is a measure one set of indestructible weakly compact cardinals below this measurable cardinal. Indestructible weak compactness gives rise to generic embeddings in a natural way, which can be used to prove results on the stationary tower and on strong forcing axioms like $\mathsf{MA}^+(\sigma-\text{closed})$.