Research Statement

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I mainly work in infinitary combinatorics, in particular

- the combinatorial structure of the universe and of inner models
- the construction of forcing notions using combinatorial principles
- applications, e.g. in topology.

In my Ph.D. thesis I gave an analysis of the combinatorial structure of inner models L[X] where X satisfies codensation, amenability and coherence. For this, I generalized Jensen's notion of higher-gap morasses to gaps of arbitrary size and proved that they in a certain sense completely capture the combinatorial structure of such models.

My present work is concerned with the construction of forcing notions along morasses. The idea is simple: I generalize the classical notion of iterated forcing with finite support. However, instead of considering linear systems of forcings and taking direct limits, I consider higher-dimensional systems and take the morass limit. Like in the case of normal finite support iterations, chain conditions are preserved.

As first applications, I constructed (1) a ccc forcing of size ω_1 that adds an ω_2-Suslin tree, (2) a ccc forcing that adds a chain $\langle X_\alpha \mid \alpha < \omega_2 \rangle$ such that $X_\alpha \subseteq \omega_1$, $X_\beta - X_\alpha$ is finite and $X_\alpha - X_\beta$ has size ω_1 for all $\alpha < \beta < \omega_2$ and (3) a ccc forcing of size ω_1 that adds a 0-dimensional T_2 topology on ω_3 which has spread ω_1.

Applications (1) and (2) use two-dimensional systems, i.e. gap-1 morasses, while application (3) uses a three-dimensional system, i.e. a gap-2 morass. That (2) holds, was first shown by Piotr Koszmider with Todorcevic's method of rho-functions. However, in (3) even the consistency of the existence of such a topology was open.

In preperation: It is consistent that there exists a function $g : [\omega_3]^2 \to \omega_1$ such that $\{\xi < \alpha \mid g(\xi, \alpha) = g(\xi, \beta)\}$ is finite for all $\alpha < \beta < \omega_3$.

Related results are contained in S. Todorcevic's book "Walks on ordinals and their characteristics".