Models of Set Theory I. - Summer 2019

PD Dr. Philipp Lücke Problem sheet 6

Problem 21 (4 Points)**.** Prove Proposition 2.2.1. from the lecture course: Assume ZF^-. Given $a_0, a_1 \in {}^{<\omega}\mathrm{Ord}$, we define

$$\begin{aligned} a_0 \prec^* a_1 \iff & \max(\mathrm{ran}(a_0)) < \max(\mathrm{ran}(a_1)) \\ & \vee \big(\max(\mathrm{ran}(a_0)) = \max(\mathrm{ran}(a_1)) \ \wedge\ \mathrm{dom}(a_0) < \mathrm{dom}(a_1)\big) \\ & \vee \big(\max(\mathrm{ran}(a_0)) = \max(\mathrm{ran}(a_1)) \ \wedge\ \mathrm{dom}(a_0) = \mathrm{dom}(a_1) \\ & \quad \wedge\ \exists n \in \mathrm{dom}(a_0)\ [a_0 \restriction n = a_1 \restriction n \ \wedge\ a_0(n) < a_1(n)]\big). \end{aligned}$$

Then the relation $\prec^*$ strongly well-orders the class ${}^{<\omega}\mathrm{Ord}$.

Problem 22. Let $\mathrm{V} = \mathrm{HOD}$ denote the statement "$\forall x\ x \in \mathrm{HOD}$".

(1) (1 Point) Show that there is an $\mathcal{L}_\in$-formula $\varphi(v_0, v_1)$ with the property that

$$\mathrm{ZF} + \mathrm{V} = \mathrm{HOD} \vdash \text{"}\textit{The relation } \{(a,b) \mid \varphi(a,b)\} \textit{ is a well-ordering of } \mathrm{V} \textit{ of order-type } \mathrm{Ord}\text{"}.$$

(2) (3 Points) Show that

$$\mathrm{ZF} \vdash \text{"}\textit{If the relation } \{(a,b) \mid \varphi(a,b)\} \textit{ is a well-ordering of } \mathrm{V} \textit{ of order-type } \mathrm{Ord}\textit{, then } \mathrm{V} = \mathrm{HOD} \textit{ holds}\text{"}.$$

holds for every $\mathcal{L}_\in$-formula $\varphi(v_0, v_1)$.

Problem 23. Prove Proposition 3.1.1. from the lecture course:

(1) (2 Points) Assume ZF^-. For every set x, the class ${}^{<\omega}x$ is a set.

(2) (2 Points) The canonical formula defining the class function that sends a set x to ${}^{<\omega}x$ is a $\Delta_1^{\mathrm{ZF}^-}$-formula.

Problem 24 (8 Points)**.** Show $\mathrm{ZF}^- \vdash (\mathrm{ZF}^-)^{\mathrm{L}}$.

Please hand in your solutions on Monday, May 13, before the lecture.

The student council of mathematics will organize the math party on 9/05 in N8schicht. The presale will be held on Mon 6/05, Tue 7/05 and Wed 8/05 in the mensa Poppelsdorf. Further information can be found at fsmath.uni-bonn.de.