Set theory - Winter Semester 2014

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Problem 31 (8 points)**.** (1) Prove that $S \cap C$ is stationary in ω_1 for all clubs C in ω_1 and all stationary subsets S of ω_1.

(2) Suppose that S is a stationary subset of ω_1. Prove that the set S_ω of all $\gamma \in S$ with the following property is stationary:

For all $\alpha < \gamma$, there is a closed subset C of S with $\min(C) \geq \alpha$, $\mathrm{type}(C) = \omega + 1$, *and* $\sup(C) = \gamma$.

(3) Suppose that S is a stationary subset of ω_1. Prove for all ordinals $\beta < \omega_1$ by induction that the set S_β of all $\gamma \in S$ with the following property is stationary:

For all $\alpha < \gamma$, there is a closed subset C of S with $\min(C) \geq \alpha$, $\mathrm{type}(C) = \beta + 1$, *and* $\sup(C) = \gamma$.

(Hint: consider the club $\bigcap_{\bar{\beta}<\beta} \lim(S_{\bar{\beta}})$, where $\lim(A)$ denotes the set of limit points $\alpha < \omega_1$ of $A \subseteq \omega_1$, and use (1), (2).)

Problem 32 (6 points)**.** Consider a train which stops successively at every $\alpha < \omega_1$. At each stop, the following happens.

(1) First, if the train is not empty, one passenger leaves the train (we don't know which one).

(2) Second, ω many passengers get on the train.

Show that at time ω_1, the train is empty. *(Hint: Use Fodor's Lemma.)*

Problem 33 (4 points)**.** Suppose that κ is an infinite regular cardinal. Suppose that $f_\alpha \colon \kappa \to \kappa$ for each $\alpha < \kappa$, and f_α, f_β are almost disjoint for all $\alpha < \beta < \kappa$. Show that there is a function $f \colon \kappa \to \kappa$ such that f, f_α are almost disjoint for all $\alpha < \kappa$.

Due Wednesday, December 03, before the lecture.