Set theory - Winter Semester 2014

Problems	Dr. Philipp Schlicht
Series 6	Dr. Philipp Lücke

Problem 22 (6 points)**.** Prove the following equalities directly from the definition of the product, without using results about products from the lecture.

(1) $\prod_{0\neq n\in\omega} n = 2^{\aleph_0}$.
(2) $\prod_{n\in\omega} \aleph_n = (\aleph_\omega)^\omega$.

Problem 23 (4 points)**.** Determine λ^κ for all $\kappa, \lambda \in Card \setminus \omega$ assuming GCH.

Problem 24 (6 points)**.** Suppose that κ is an infinite regular cardinal. Prove the following statements.

(1) The Axiom of Choice holds in $(H_\kappa, \in)$.
(2) The statement that $|x|$ is defined for every set x holds in $(H_\kappa, \in)$.
(3) If κ is a successor cardinal, then the Power Set Axiom does not hold in $(H_\kappa, \in)$.

Problem 25 (4 points)**.** Prove in ZF that the following statement is equivalent to the Axiom of Choice: *For all sets x, y and every surjective function $f\colon x \to y$, there is a function $g\colon y \to x$ with $f \circ g = \mathrm{id}_y$.*

Due Wednesday, November 19, before the lecture.