# Locally moving groups and the reconstruction of structures from their automorphism groups

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Let $X$ be a regular space and $G$ be a group of auto-homeomorphisms of $X$. $G$ is said to be a locally moving group (LM group) for $X$, if for every nonempty open $U \subseteq X$, there is $g \in G \setminus \{\mathrm{Id}\}$ such that $G \restriction (X \setminus U) = \mathrm{Id}$. Let $\mathrm{Ro}(X)$ denote the partially ordered set of regular open subsets of $X$. (A subset of $X$ is regular open, if it is equal to the interior of its closure.)

**Theorem** Let $X, Y$ be regular spaces and $G, H$ be LM groups for $X$ and $Y$. Suppose that $\phi$ is an isomorphism between $G$ and $H$ (as abstract groups). Then there is an isomorphism $\psi$ between $\mathrm{Ro}(X)$ and $\mathrm{Ro}(Y)$ such that for every $g \in G$, $\phi(g) = \psi \circ g \circ \psi^{-1}$.

The above theorem has many applications in answering questions of the type: "When does the automorphism group of a mathematical structure determine the structure". For example:

**Theorem** Let $U, V$ be open subsets of normed spaces $E$ and $F$. Suppose that $\phi$ is an isomorphism between the group $\mathrm{H}(X)$ of all auto-homeomorphisms of $X$, and the group $\mathrm{H}(Y)$ of all auto-homeomorphisms of $Y$. Then there is a homeomorphism $\tau$ between $X$ and $Y$ such that for every $g \in \mathrm{H}(X)$, $\phi(g) = \tau \circ g \circ \tau^{-1}$.

I shall describe this framework and some results. I shall also mention some open questions in this area.