Models of Set Theory I - Summer 2013

Prof. Dr. Peter Koepke, Dr. Philipp Lücke Problem sheet 4

Problem 14 (4 Points)**.** Let $\mathbb{R}$ denote the set of real numbers (constructed as Dedekind-cuts of $(\mathbb{Q}, <)$) and M be a transitive set with ZFC^M. Show: if $\mathbb{R}^M$ denotes the set of real numbers constructed in M, then $\mathbb{R}^M$ is a subset of $\mathbb{R}$ that is dense with respect to the natural ordering of the reals.

Problem 15 (6 Points)**.** Let $\varphi(v_0, v_1)$ be a definite formula.

(1) Show that the formula $\exists x\ \varphi(v_0, x)$ is $\mathrm{H}(\kappa)$-absolute for every uncountable regular cardinal κ (Hint: *Use Skolem hulls and Mostowski collapses*).

(2) Show that the formula $\exists x\ \varphi(v_0, x)$ is $\mathrm{H}(\kappa)$-absolute for every uncountable cardinal κ with $(\mathrm{ZF}^-)^{\mathrm{H}_\kappa}$.

Problem 16 (6 Points)**.** Let $\varphi(v)$ be the canonical $\in$-formula defining the class of inaccessible cardinals.

(1) Show that φ is V_α-absolute for every $\alpha \in \mathrm{Ord}$.

(2) Show: if M and N are transitive sets with ZFC^M, ZFC^N and $M \subseteq N$, then

$$\langle N, \in \rangle \models \varphi(\kappa) \ \longrightarrow \langle M, \in \rangle \models \varphi(\kappa)$$

for all $\kappa \in M$.

(3) Show: if ZFC is consistent, then $\mathrm{ZFC} \nvdash \exists \kappa\ \varphi(\kappa)$.

Problem 17 (4 Points)**.** Let $\varphi(v)$ be the canonical $\in$-formula defining the cardinal $\aleph_1$. Show that $\varphi(v)$ is not definite (Hint: *Consider the model* $\mathrm{H}_{\aleph_2}$ *and form the Skolem hull of* $\{\aleph_1\}$).

Please hand in your solutions on Wednesday, May 08 before the lecture.