FREE GROUPS AND AUTOMORPHISM GROUPS OF INFINITE FIELDS

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ABSTRACT. We consider the following question: is there a field K whose automorphism group is a free group of cardinality greater than the cardinality of K? Saharon Shelah showed that such a field cannot be countable. On the other hand, Wilfried Just, Saharon Shelah and Simon Thomas proved that the existence of such a field is consistent with the axioms of set theory.

In this talk, I want to present a result that implies a positive answer to the above question: if λ is a cardinal such that the set of all countable subsets of λ has cardinality λ and p is either 0 or a prime number, then there is a field of characteristic p and cardinality λ whose automorphism group is a free group of cardinality 2^{λ} .

The methods developed in the proof of this result also allow us to show that the above cardinal arithmetic assumption is consistently not necessary for the existence of such fields and that it is necessary to use large cardinal assumptions to construct a model of set theory containing a cardinal λ of uncountable cofinality with the property that no free group of cardinality greater than λ is isomorphic to the automorphism group of a field of cardinality λ .

The results in this talk are joint work with Saharon Shelah.

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