Mathematisches Institüt der Universität Bonn
winter semester 2010-11
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Formal derivations and natural proofs exercise sheet 1

1. In each of the three calculuses we defined, prove that $\forall x\phi \rightarrow \exists x\phi$ is a tautology. Moreover, in the sequent calculus show that $\phi \wedge \psi \rightarrow \phi$ and $((\phi \vee \psi) \rightarrow \chi) \rightarrow (\phi \rightarrow \chi)$ are tautologies.

2. In the sequent calculus, prove that $\equiv$ is an equivalence relation.

3. There is an photocopied attachment, pages 102–105 of the "Collected works of Kurt Gödel, volume I", in which a *functional calculus* is introduced. Keeping in mind that (x) means $\forall x$, and (Ex) means $\exists x$ in the notation of the time, write down the calculus in modern notation and prove (1) and (2) of page 105 (page 104 in German).