DESCRIPTIVE SET THEORY AT UNCOUNTABLE CARDINALS: $\mathbf{\Delta}^1_1$-SUBSETS OF ${}^\kappa\kappa$

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Abstract. Let κ be an uncountable regular cardinal with $\kappa = \kappa^{<\kappa}$. A subset of $({}^\kappa\kappa)^n$ is a $\mathbf{\Sigma}^1_1$-subset if it is the projection $\rho[T]$ of all cofinal branches through a κ-tree T on κ^{n+1}. We define $\mathbf{\Sigma}^1_k$-, $\mathbf{\Pi}^1_k$- and $\mathbf{\Delta}^1_k$-subsets of $({}^\kappa\kappa)^n$ as usual.

Given an arbitrary subset A of ${}^\kappa\kappa$, I showed that there is a $<\kappa$-closed forcing $\mathbb{P}$ that satisfies the κ^+-chain condition and forces A to be a $\mathbf{\Delta}^1_1$-subset of ${}^\kappa\kappa$ in every $\mathbb{P}$-generic extension of V. This result allows us to construct a forcing with the above properties that forces the existence of a well-ordering of ${}^\kappa\kappa$ whose graph is a $\mathbf{\Delta}^1_2$-subset of ${}^\kappa\kappa \times {}^\kappa\kappa$. If we also assume $2^\kappa = \kappa^+$, then we can produce a generic well-ordering of ${}^\kappa\kappa$ whose graph is a $\mathbf{\Delta}^1_1$-subset of ${}^\kappa\kappa \times {}^\kappa\kappa$.

In my talk, I want to present the central ideas behind the proofs of these results, focusing on coding subsets of ${}^\kappa\kappa$ by κ-Kurepa trees in forcing extensions and the strong absoluteness properties of this coding.