

Singularizing Successor Cardinals by Forcing

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October 19, 2010

Namba forcing singularizes the successor of $\kappa = \aleph_1$ without collapsing κ . In my talk I study the case $\kappa > \aleph_1$:

Theorem 1. Let μ be a measurable cardinal and let κ be a regular cardinal such that $\aleph_1 < \kappa < \mu$. Then there are successive forcing extensions $V \subseteq M \subseteq N$ such that

$$\kappa \in \text{Card}^N, \kappa > \aleph_1^N, \text{ and } \text{cof}^N(\kappa^{+M}) = \aleph_0.$$

The construction combines Prikry forcing for μ with Levy collapsing μ to κ^+ . Conversely, standard covering arguments show that singularizing larger successor cardinals indeed requires the strength of measurable cardinals.

This work is currently being extended to uncountable cofinalities together with Dominik Adolf at Münster.