NEW COMBINATORIAL PRINCIPLE ON SINGULAR CARDINALS

TOSHIMICHI USUBA

We introduce a new combinatorial principle UB_λ, UnBranched property, for a singular cardinal λ. UB_λ is the principle asserting that there exists a function $f : [\lambda^+]^{<\omega} \to \lambda^+$ such that for every f-closed $x, y \subseteq \lambda^+$, if $x \cap \lambda = y \cap \lambda$, $\sup(x \cap \lambda) = \lambda$, and $\sup(x) \leq \sup(y)$ then $x \subseteq y$. We show that UB_λ is implied by the principle ADS_λ. ADS_λ is known as a very weak principle, but is inconsistent with some large cardinals. However we also show that UB_λ is consistent with almost all large cardinals and large cardinal properties: For instance, UB_λ is consistent with the existence of supercompact cardinals below λ and with Martin's Maximum. We observe some applications of UB_λ such as:

(1) $\mathrm{UB}_{\omega_\omega}$ refutes $\langle \aleph_{\omega+1}, \aleph_\omega \rangle \twoheadrightarrow \langle \aleph_2, \aleph_1 \rangle$ (but $\mathrm{UB}_{\omega_\omega}$ is consistent with $\langle \aleph_{\omega+1}, \aleph_\omega \rangle \twoheadrightarrow \langle \aleph_1, \aleph_0 \rangle$).
(2) UB_λ implies a kind of weak covering lemma.
(3) UB_λ implies that for a regular κ with $\mathrm{cf}(\lambda) < \kappa$, every normal ideal over $P_\kappa \lambda$ which has no λ^+ disjoint positive sets is λ^+-saturated.

E-mail address: toshimichi.usuba@hcm.uni-bonn.de