Mathematisches Institut
der Universität Bonn
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Winter 09/10
November 23, 2009
Exercise Sheet 6

Exercises for

Models of Set Theory II

21. Let $\mathcal{I}$ be a σ-complete ideal containing all singletons. Prove:
(a) $\aleph_1 \leq add(\mathcal{I}) \leq cov(\mathcal{I}) \leq cof(\mathcal{I})$
(b) $\aleph_1 \leq add(\mathcal{I}) \leq non(\mathcal{I}) \leq cof(\mathcal{I})$.

22. (a) $add(\mathcal{I})$ is regular
(b) $cf(non(\mathcal{I})) \geq add(\mathcal{I})$
(c) $cf(cof(\mathcal{I})) \geq add(\mathcal{I})$.

23. Show that the following statements are equivalent:
(a) $cov(\mathcal{M}) > \kappa$
(b) For every family $\{D_\alpha \mid \alpha < \kappa\}$ of dense open subsets of $\mathbb{R}$ there exists a countable subset $X \subseteq \mathbb{R}$ such that $|X - D_\alpha| < \aleph_0$ for all $\alpha < \kappa$.

24. Show that the following statements are equivalent:
(a) $cov(\mathcal{M}) > \kappa$
(b) MA_κ holds for countable forcings, i.e. if $\mathbb{Q}$ is a countable forcing and $\mathfrak{D}$ is a family of κ-many in $\mathbb{Q}$ dense sets then there exists a filter H on $\mathbb{Q}$ such that $D \cap H \neq \emptyset$ for all $D \in \mathfrak{D}$.
Hint: Use exercise 34 of Models of Set Theory I.

Every problem will be graded with 8 points.

Please hand in your solutions during the lecture at November 30, 2009.