Mathematisches Institut
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Winter 09/10
November 2, 2009
Exercise Sheet 3

Exercises for

Models of Set Theory II

9. Let $\mathbb{P}$ be a forcing and $p \in \mathbb{P}$. Assume $p \Vdash \exists x\ \varphi(x)$. Show that there exists a name $\dot{x}$ such that $p \Vdash \varphi(\dot{x})$.

For regular $\kappa \geq \omega$ let $\mathbb{C}(\kappa) = \{p \mid \exists \alpha < \kappa\ p : \alpha \to 2\}$ ordered by $p \leq q$ iff $p \supseteq q$. Let M be a ground model which satisfies GCH.

10. Show that in M the following holds: There exist sequences $\langle \dot{\mathbb{Q}}_i \mid i < \omega \rangle$ and $\langle \mathbb{P}_i \mid i \leq \omega \rangle$ such that $\langle \mathbb{P}_i \mid i \leq \omega \rangle$ is the finite support iteration of $\langle \dot{\mathbb{Q}}_i \mid i < \omega \rangle$ and $1_{\mathbb{P}_n} \Vdash_{\mathbb{P}_n} \dot{\mathbb{Q}}_n = \mathbb{C}(\omega_n)$ for all $n \in \omega$.

11. Let G be $\mathbb{P}_\omega$-generic over M. Prove for all $i < \omega$:

(a) $M[G_i] \models GCH$

(b) $Card^{M_i} = Card^M$.

12. Let $\kappa_n = \omega_n^M$. Prove for all $n \in \omega$ that $M[G] \models cf(\kappa_n) = \omega$ and hence $M[G] \models card(\kappa_n) = \omega$.

Hint: Consider the generic functions $f_i : \kappa_i \to 2$. For $n \leq i$ let

$$a_i = min\{\delta < \kappa_n \mid f_i(\delta) = 1\}.$$

Then $\langle a_i \mid i \geq n \rangle$ is cofinal in κ_n.

Every problem will be graded with 8 points.

Please hand in your solutions during the lecture at November 9, 2009.