Mathematisches Institut
der Universität Bonn
P. Koepke, B. Irrgang

Summer Semester 2009
June 29, 2009
Exercise Sheet 9

Exercises for

Models of Set Theory I

33. Let M be a ground model and $\mathbb{P}, \mathbb{Q} \in M$ be forcings. A map $\pi : \mathbb{P} \to \mathbb{Q}$ is called dense embedding if

(i) $\forall p, q \in \mathbb{P}\ (p \leq q \ \rightarrow \pi(p) \leq \pi(q))$

(ii) $\forall p, q \in \mathbb{P}\ (p, q$ incompatible $\rightarrow \pi(p), \pi(q)$ incompatible$)$

(iii) $\pi[\mathbb{P}]$ is dense in $\mathbb{Q}$.

Let $\pi : \mathbb{P} \to \mathbb{Q}$, $\pi \in M$ be a dense embedding and G be M-generic on $\mathbb{P}$. Show that $\{q \in \mathbb{Q} \mid \exists p \in G\ \pi(p) \leq q\}$ is M-generic on $\mathbb{Q}$. Moreover, prove conversely that $\pi^{-1}[H]$ is M-generic on $\mathbb{P}$ if H is M-generic on $\mathbb{Q}$. That is, $\mathbb{P}$ and $\mathbb{Q}$ yield the same generic extensions.

34. Let $\mathbb{P} = \{p : n \to \omega \mid n \in \omega\}$ ordered by the reversed subset relation and $\mathbb{Q}$ be any countable forcing such that $\forall q \in \mathbb{Q}\ \exists q_1, q_2 \leq q\ (q_1 \perp q_2)$. Show that there exists a dense embedding $\pi : \mathbb{P} \to \mathbb{Q}$.

35. Show that $\forall p, q \in \mathbb{P}\ (p \leq q \ \leftrightarrow \ p \Vdash \check{q} \in \dot{G})$ if $\mathbb{P}$ iff

$$\forall p, q \in \mathbb{P}\ (p \not\leq q \ \rightarrow \ \exists r \leq p\ (r \perp q)).$$

36. Let M be a ground model. Show that there exists $\mathbb{P} \in M$ such that $\omega_1^M < \omega_1^{M[G]}$.

Hint: Consider the finite functions $f \in M$ with $dom(f) \subseteq \omega$ and $rng(f) \subseteq \omega_1^M$.

Every problem will be graded with 8 points.

Please hand in your solutions during the lecture at July 6, 2009.