Mathematisches Institut
der Universität Bonn
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Summer Semester 2009
June 15, 2009
Exercise Sheet 7

Exercises for

Models of Set Theory I

Let M be a ground model and $\mathbb{P} \in M$ be a forcing.

25. Let $1_{\mathbb{P}} \in G \subseteq \mathbb{P}$ be such that $\forall p \in G\ \forall q \in \mathbb{P}\ (q \geq p \rightarrow q \in G)$. Assume moreover that $G \cap D \neq \emptyset$ for all sets $D \in M$ which are dense in $\mathbb{P}$. Show that

$$\forall p, q \in G\ \exists r \in \mathbb{P}\ r \leq p, q$$

implies that G is a M-generic filter on $\mathbb{P}$. That is, it suffices to find $r \leq p, q$ in $\mathbb{P}$ instead of G.

26. Show that a filter G on $\mathbb{P}$ is M-generic if and only if $G \cap E \neq \emptyset$ whenever $E \in M$ and $\forall p \in \mathbb{P}\ \exists q \in E$ (p and q are compatible). In particular, G is M-generic if and only if $G \cap A \neq \emptyset$ for every maximal antichain $A \subseteq \mathbb{P}$ with $A \in M$.

27. Let $\dot{x}, \dot{y} \in M$ be names such that $dom(\dot{x}), dom(\dot{y}) \subseteq \{\check{n} \mid n \in \omega\}$. Let

$$\dot{c} = \{(\check{n}, p) \mid \exists q, r\ (p \leq q, r\ \wedge\ (\check{n}, q) \in \dot{x}\ \wedge\ (\check{n}, r) \in \dot{y})\}$$

$$\dot{e} = \{(\check{n}, p) \mid \forall q \in \mathbb{P}\ (\ (\check{n}, q) \in \dot{x}\ \rightarrow\ p, q \text{ incompatible})\}.$$

Show that $\dot{c}^G = \dot{x}^G \cap \dot{y}^G$ and $\dot{e}^G = \omega - \dot{x}^G$ for all M-generic G.

28. Let $p, q \in \mathbb{P}$ be incompatible. Show that

$$\{\dot{x} \in M \mid p \Vdash \dot{x} = \check{\emptyset}\}$$

is a proper class in M.

Every problem will be graded with 8 points.

Please hand in your solutions during the lecture at June 22, 2009.