

Exercises for  
Models of Set Theory I

Let in the following  $M$  be a ground model,  $(\mathbb{P}, \leq_{\mathbb{P}}, 1_{\mathbb{P}}) \in M$  be a forcing and  $G$  be a  $M$ -generic filter on  $\mathbb{P}$ .

21. (a) Assume that  $x, y \in M[G]$ . Prove that also  $x \times y \in M[G]$ .  
(b) Let  $\dot{x}, \dot{y} \in M$  be names for  $x, y \in M[G]$ . Define a name  $\dot{z} \in M$  for  $(x, y) \in M[G]$ .
22. Assume that  $G$  is a filter on  $\mathbb{P}$ , which is not necessarily generic. Prove that  $M[G] = M$  if  $G \in M$ .
23. Let  $\mathbb{P}$  be Cohen forcing. Prove that there exists in  $M[G]$  an  $a \subseteq \omega$  such that  $a \notin M$ .
24. Let  $D \subseteq \mathbb{P}$ ,  $D \in M$  and  $p \in G$ . Assume that  $\forall q \leq p \exists r \in D$   $r, q$  compatible. Show that  $G \cap D \neq \emptyset$ .

Every problem will be graded with 8 points.

Please hand in your solutions during the lecture at June 15, 2009.