Mathematisches Institut
der Universität Bonn
P. Koepke, B. Irrgang

Summer Semester 2009
May 20, 2009
Exercise Sheet 5

Exercises for

Models of Set Theory I

17. Prove

$$(V = HOD)^L.$$

18. For cardinals $\kappa \geq \omega$ let $H_\kappa = \{x \mid card(TC(x)) < \kappa\}$. Prove that

$$L_\kappa = (H_\kappa)^L$$

for all cardinals $\kappa \geq \omega$.

Hint: Argue similar to lemma 62.

19. Assume $V = L$. Let $M = L_{\omega_2}$ and $N \subseteq M$ be such that every $\in$-formula φ is N-M-absolute. Show that $N \cap \omega_1 \in Ord$.

Hint: For $\alpha \in N \cap \omega_1$ there exists $f \in L_{\omega_2}$ such that $f : \omega \to \alpha$ is surjective. Hence $rng(f) = \alpha \subseteq M$.

20. For finite functions $a : dom(a) \to Ord$, $dom(a) \subseteq \omega$ define $a < b$ iff

$$max(dom(a)) < max(dom(b)) \quad or$$

$$(\; max(dom(a)) = max(dom(b))$$

$$\wedge \; \exists n \in dom(b)(a \upharpoonright n = b \upharpoonright n \wedge (n \in dom(a) \to a(n) < b(n)))).$$

Prove:

(a) $<$ is a wellorder.

(b) $\{a \mid a < b\} \in V$ for all b.

Every problem will be graded with 8 points.

Please hand in your solutions during the lecture at June 8, 2009.