Mathematisches Institut
der Universität Bonn
P. Koepke, B. Irrgang

Summer Semester 2009
May 6, 2009
Exercise Sheet 3

Exercises for

# Models of Set Theory I

9. Define a relation $\in'$ on $\mathbb{N}$ by

$$m \in' n \quad \text{iff} \quad \exists s, r \in \mathbb{N}(n = 2^{m+1}s + 2^m + r \wedge r < 2^m).$$

(a) Which axioms of set theory hold in this structure?

(b) What do the ordinals of this structure look like?

10. Prove that for every transitive $ZF^-$-model $M$, $V_\alpha^M = V_\alpha \cap M$ for all $\alpha \in M$.

11. Let $M$ be a transitive $ZF^-$-model. Let $(X, E) \in M$ be a well-founded, extensional relation and $\pi : (X, E) \cong (N, \in)$ the Mostowski collapse. Show that $\pi, N \in M$.

12. Prove: If $M$ and $N$ are two transitive models of $ZFC$ with the same sets of ordinals, i.e. $\mathfrak{P}^M(Ord^M) = \mathfrak{P}^N(Ord^N)$, then $M = N$.
Hint: Show that $V_\alpha^M = V_\alpha^N$ for all $\alpha \in Ord^M = Ord^N$, using exercise 11.

Every problem will be graded with 8 points.

Please hand in your solutions during the lecture at May 13, 2009.