Mathematisches Institut
der Universität Bonn
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Summer Semester 2009
July 13, 2009
Exercise Sheet 11

Exercises for

Models of Set Theory I

41.(Continuation of exercise 40) Consider the forcing $\mathbb{P}$ in some ground model M. Let G be M-generic on $\mathbb{P}$ and $T = \bigcup\{p \mid p \in G\}$. Show that T is a Suslin tree in $M[G]$.

Hint: To show that T has no uncountable antichain, let $p \Vdash (\langle x_\eta \mid \eta \in \omega_1\rangle$ enumerates an antichain in T). For every $\eta \in \omega_1$ pick $p_\eta \leq p$ and $\alpha_\eta \in \omega_1$ such that $p_\eta \Vdash x_\eta = \check{\alpha}_\eta$. Then there exist $\eta \neq \gamma$ and $q \leq p_\eta, p_\gamma$ such that $q \Vdash \check{\alpha}_\eta \leq_T \check{\alpha}_\gamma$. Contradiction.

Let $M \models GCH$ be a ground model. Let $\kappa > \omega$ be regular in M and $\mathbb{P} = Fn(\kappa \times \omega, 2, \omega)^M$ be the forcing which adds κ Cohen reals. Let G be M-generic on $\mathbb{P}$. For $\gamma < \kappa$ let $\mathbb{P}_\gamma = Fn(\gamma \times \omega, 2, \omega)^M$ and $G_\gamma = \{p \in \mathbb{P}_\gamma \mid p \in G\}$. Let $\mathbb{Q} = (\omega, \leq_\mathbb{Q})$ be a countable forcing in $M[G]$.

42. Show that there exists a $\gamma < \kappa$ and a $\mathbb{P}_\gamma$-name $\dot{x}$ such that $\dot{x}^{G_\gamma} = \mathbb{Q}$.

43. Show that if $\mathfrak{D}$ is in $M[G]$ a family of $< \kappa$ many in $\mathbb{Q}$ dense sets then there exists a $\gamma < \kappa$ and a $\mathbb{P}_\gamma$-name $\dot{y}$ such that $\dot{y}^{G_\gamma} = \mathfrak{D}$.

44. Show that MA holds in $M[G]$ for countable forcings, i.e. if $\mathbb{Q}$ is a countable forcing in $M[G]$ and $\mathfrak{D}$ is in $M[G]$ a family of $< \kappa$ many in $\mathbb{Q}$ dense sets then there exists in $M[G]$ a filter H on $\mathbb{Q}$ such that $D \cap H \neq \emptyset$ for all $D \in \mathfrak{D}$.

Hint: Exercise 34 and product lemma.

Every problem will be graded with 8 points.

Please hand in your solutions during the lecture at July 20, 2009.