Mathematisches Institut
der Universität Bonn
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Summer Semester 2009
July 6, 2009
Exercise Sheet 10

Exercises for

Models of Set Theory I

37. Drop in the definition of MA the condition that the forcings are ccc. Show that the resulting statement is false if $2^\omega > \omega_1$.

38. Let $\lambda > \omega$ be a singular cardinal. Show that if a forcing $\mathbb{P}$ is λ-closed then it is even λ^+-closed.

39. Let T be a Suslin tree such that every $x \in T$ has two immediate successors p_x, q_x. Let $\mathbb{P}$ be the forcing defined in problem 31. Prove that $\mathbb{P} \times \mathbb{P}$ does not satisfy ccc.

Remark: The existence of Suslin trees is consistent. Hence the exercise shows that the existence of a ccc forcing $\mathbb{P}$ such that $\mathbb{P} \times \mathbb{P}$ does not satisfy ccc is consistent.

40. Let $\mathbb{P}$ be the forcing consisting of finite trees $(T, <_T)$ such that $T \subseteq \omega_1$, and such that $\alpha < \beta$ if $\alpha <_T \beta$; $(T_1, <_{T_1})$ is stronger than $(T_2, <_{T_2})$ if and only if $T_1 \supseteq T_2$ and $<_{T_2} = <_{T_1} \upharpoonright T_2$.

Prove that $\mathbb{P}$ satisfies ccc.

Every problem will be graded with 8 points.

Please hand in your solutions during the lecture at July 13, 2009.