Mathematisches Institut
der Universität Bonn
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Summer Semester 2009
April 22, 2009
Exercise Sheet 1

Exercises for

Models of Set Theory I

1. Let $H_\kappa = \{x \mid card(TC(x)) < \kappa\}$. Prove that φ^{H_κ} holds for all φ of ZFC minus the power set axiom if κ is a regular uncountable cardinal.

2. Let $M = \{x \mid \varphi(x, a_1, \ldots, a_n)\}$ and W be classes. Define $M^W := \{x \in W \mid \varphi^W(x, a_1, \ldots, a_n)\}$. Prove that if $W = V_{\omega+1}$ then
(a) $\omega^W = \omega$.
(b) (x is finite)^W if and only if x is finite.

3. Let W be transitive and $x \in W$. Prove that
(a) $(\bigcup x)^W = \bigcup x$
(b) $(\mathfrak{P}(x))^W = \mathfrak{P}(x) \cap W$.

4. Prove that in ZF minus Regularity, φ^V holds for every axiom φ of ZF. Hence if ZF minus Regularity is consistent, then also ZF is consistent.

Every problem will be graded with 8 points.

Please hand in your solutions during the lecture at April 29, 2009.