Young set theory workshop

Haus Annaberg in Bonn, 21-25 January 2008

Bonn Mathematical Logic Group

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HIM HAUSDORFF RESEARCH INSTITUTE FOR MATHEMATICS

bonn international graduate school in mathematics

hausdorff center for mathematics

Young researchers in set theory workshop

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We organised the 'Young Researchers in Set Theory' meeting to give talented young researchers in set theory the opportunity to learn from experts and from each other in a friendly co-operative environment.

We used the term 'young' to mean a PhD student or a post-doc (i.e., someone without a permanent position) and the term 'senior' for a researcher with a permanent position.

With this workshop we aimed also to create a network of young set theorists and senior researchers who support their work in order to establish working contacts and to better disseminate knowledge in the field. We feel that this goal has been achieved.

The workshop took place in Haus Annaberg in Bonn. This is a conference center situated on an idyllic forest covered hill about twenty minutes from the center of Bonn. All areas of Haus Annaberg that were given to the workshop have been used by the participants for lively set theory discussions.

We are happy to report that the workshop has been a success even greater than we expected.

The discussion sessions were very lively, spontaneous and a significant amount of them went well into the night. The tutorial speakers were always present and willing to answer questions about their tutorials and to help with other problems of the participants. Other senior researchers joined in as well and were always very helpful. Several collaborations have been formed.

Our participants have shown great interest in the repetition of such an event.

This workshop is funded by BIGS (Bonn International Graduate School) and HCM (Hausdorff Center for Mathematics), with the organisatorial help of HIM (Hausdorff Research Institute for Mathematics). We thank these three institutions for their support.

Ioanna Dimitriou,
Bernhard Irrgang,
Katie Thompson and
Jip Veldman

Contents

Abstracts for tutorials

The abstracts are arranged alphabetically by first name.

The axiom of determinacy and the Wadge hierarchy

Alessandro Andretta
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After surveying some basic facts about the axiom of determinacy, I will focus on the Wadge hierarchy and its properties. If time permits, I will try to prove a few results on cardinalities in the AD-world.

Real forcing

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The first application of forcing was the consistency proof of ¬CH. The forcing notion that we now call "Cohen forcing" adds a large number (at least $\aleph_2$) of new reals without collapsing cardinals.

Since then, an almost uncountable number of forcing notions adding reals has been invented (Solovay=random, Sacks=perfect, Miller=superperfect, etc), plus a few methods (product, composition, iteration, amalgamation) of combining these forcing notions.

When designing a forcing notion to solve a specific problem, one usually has to take care of the following two aspects:

(1) The forcing notion has to add a new object g (often a real number) satisfying some property X
(as: g is faster, higher, stronger than all reals of the ground model).

(2) The forcing notion should not add objects/reals r with some property Y
(such as: r codes a well-order of ω of type ω_1^V, r destroys/trivialises this or that structure from the ground model).

In my tutorial I will give many examples for forcing notions adding reals, and explain why they add reals with some property X, and also (what is often more difficult) why they do not add reals with property Y. An important ingredient in such proofs are "preservation theorems", i.e., theorems of the form:

> Whenever forcing notions $P_1, P_2, \ldots$ are of a particularly nice form, then also the product/iteration/etc of these forcings has nice properties.

Set theory and model-theoretic logics

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One of the roles of logic is to serve as a tool for the study of structures. The best known tool, first order logic, cannot distinguish between cardinalities of infinite models. There are many extensions of first order logic where such and other sharper distinctions are possible. The most notable ones are the infinitary logics, logics with generalized quantifiers, and higher order logics. There are also intermediate logics which do not fit well into these three categories, such as the equicardinality quantifier "there are as many x with $\phi(x)$ as there are y with $\psi(y)$". It is an example of a *strong* logic, that is, a logic which has enough power to express properties of not only this or that model, but of the underlying set theoretical universe. The opposite is an *absolute* logic, that is, a logic the truth definition of which makes no reference to what kind of set there exists in the underlying universe.

In the tutorial I give an introduction to what is known about strong logics. What are strong logics, what are they good for, and how do they depend on set theoretical properties of the universe? What kind of compactness properties do they have? What kind of interpolation theorems? What kind of Löwenheim- Skolem properties? I will discuss in depth (1) the transfinite Ehrenfeucht-Fraïssé game, and (2) the equicardinality quantifier, both from a set theoretic point of view; methods, results and open problems.

Abstracts for talks

Is there weak compactness without inaccessibility?

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Weak compactness can be characterised using elementary embeddings or reflection of Π^1_1-sentences in a way that makes it possible to drop inaccessibility from the characterisation. Many arguments leading to results about the weakly compact ideal rely on inaccessibility. But the corresponding ideal may also be defined on cardinals that are not inaccessible. We shall discuss aspects of weak compactness that do not need inaccessibility but also facts that point to inaccessibility being an important ingredient of weak compactness.

Diamond on successor cardinals

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We give a fully-detailed blackboard proof of Shelah's remarkable theorem that $2^\lambda = \lambda^+$ is equivalent to $\diamondsuit_{\lambda^+}$ for every uncountable cardinal λ.

The original proof appears in Shelah's paper #922, and the simplified presentation we give is due to Péter Komjáth.

Large cardinals and locally defined well–orders of the universe

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I am going to present a proof of the following theorem.

Theorem 0.1. *(GCH) There is a formula $\Phi(x, y)$ without parameters and there is a definable class–sized partial order $\mathcal{P}$ preserving ZFC, GCH and cofinalities, and such that*

(1) *$\mathcal{P}$ forces that there is a well–order $\leq$ of the universe such that*

$$\{(a, b) \in H(\kappa^+)^2 : \langle H(\kappa^+), \in\rangle \models \Phi(a, b)\}$$

is the restriction $\leq\restriction H(\kappa^+)^2$ and is a well–order of $H(\kappa^+)$ whenever $\kappa \geq {}_2$ is a regular cardinal, and

(2) *for all regular cardinals $\kappa \leq$, if κ is a –supercompact cardinal in V, then κ remains –supercompact after forcing with $\mathcal{P}$.*

One key task (Task 1) in the proof of Theorem 0.1 is this: For a fixed regular cardinal $\kappa \geq {}_2$, we build a forcing iteration for manipulating certain weak guessing properties for club–sequences defined on stationary subsets of κ, in such a way that (a certain definable subset of) the set of ordinals τ for which there is some club–sequence on κ of height τ and satisfying the property codes any prescribed subset A of κ.[1]

Another task (Task 2) is the following: For the same fixed κ, given a function $F : \kappa \longrightarrow \mathcal{P}(\kappa)$ and a sequence $\mathcal{S} = \langle S_i : i < \kappa \rangle$ of pairwise disjoint stationary subsets of κ, we force in such a way that every $B \subseteq \kappa$ gets coded by some ordinal in δ^+ with respect to F and $\mathcal{S}$. This means that there is a club $E \subseteq \mathcal{P}_\kappa(\delta)$ such that for every $X \in E$ and every $i < \kappa$, if $X \cap \kappa \in S_i$, then $ot(X) \in F(X \cap \kappa)$ if and only if $i \in B$.

It is possible to add F and $\mathcal{S}$ as above, then to pick a subset A of κ coding F and $\mathcal{S}$, and then to perform Tasks 1 and 2 simultaneously, for A and for F and $\mathcal{S}$, by a nicely behaved[2] forcing. This is the *one–step construction at κ.*

The forcing $\mathcal{P}$ can be roughly described as a two–step iteration $\mathcal{B} * \dot{\mathcal{C}}$ in which $\mathcal{B}$ is a forcing iteration of length Ord adding a system of bookkeeping functions and $\dot{\mathcal{C}}$ is another iteration on which we force with the one–step forcing at κ for all the relevant κ (using the bookkeeping functions added by $\mathcal{B}$).

I intend to present the above one–step construction with some detail and to outline the general lifting lemma that we use in the large cardinal preservation part of the proof.

Aspects of indestructible weak compactness

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I will describe connections between the large cardinal phenomenon of indestructible weak compactness and closed maximality principles in order to motivate the usefulness of this concept as a measure of consistency strength. I will also give some applications of generic embeddings derived from indestructible weak compactness to stationary tower forcing and forcing axioms.

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[1] A club–sequence $\langle C_\alpha : \alpha \in dom(\vec{C}) \rangle$ has height τ iff $ot(C_\alpha) = \tau$ for all $\alpha \in dom(\vec{C})$.

[2] κ–strategically closed and κ–c.c.

Weak diamonds and the club principle

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This talk will be about my current work on weak diamonds, which is a continuation of [2] and [1]. We show that some weakenings of the club principle do not imply the existence of a Souslin tree. We show that $\diamondsuit(2^\omega, [\omega]^\omega, \text{ is constant on})$ together with CH and "all Aronszajn trees are special" is consistent relative to ZFC. This implies the analogous result for a double weakening of the club principle.

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Variations of the Mitchell model

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In a now classical construction, W. Mitchell constructed a model with no Aronszajn trees on ω_2. Recently I constructed a model in which the notions of "internally club" and "internally approachable" for structures of size $\aleph_1$ are distinct. The proof of the latter bears some resemblance to the former. I present a general framework which includes both as special cases, together with several other related combinatorial properties of ω_2, and allows these consistency results to propagate in some form to higher cardinals.

Some applications of reflection principles to pcf theory

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We present some application of reflection principles to the analysis of the partial order of reduced product of regular cardinal. The guiding example being the study of the partial order $(\prod_n \aleph_n, <^*)$, where $f <^* g$ if $f(n) \geq g(n)$ for finitely many n. The main result is that a reflection principle on $\aleph_2$ which is equiconsistent with $\aleph_2$ being weakly compact in L and which follows from Martin's maximum implies that club many points of cofinality $\aleph_2$ below $\aleph_{\omega+1}$ are approachable. This is obtained by a combination of two theorems: one by me and the other by Assaf Sharon. We then link these results to the study of the transfer principles

$$(\aleph_{\omega+1}, \aleph_\omega) \to (\aleph_{n+1}, \aleph_n).$$

In particular results of Shelah show that this Chang conjecture fails if $n > 2$. Under the assumption of our main theorem we show that it fails also for $n = 1, 2$. Levinsky Magidor and Shelah on the other hand have shown that

$$(\aleph_{\omega+1}, \aleph_\omega) \twoheadrightarrow (\aleph_1, \aleph_0)$$

is consistent.

The tree property at the double successor of a measurable

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We say that the tree property holds at a cardinal κ iff there are no κ-Aronszajn trees. We show that the following statements are equiconsistent:

(1) The tree property holds at the double successor of a measurable cardinal.
(2) There is a weakly compact hypermeasurable cardinal.

This work is joint with Sy-David Friedman.

Research statements

The research statements are arranged alphabetically by first name.

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My main research interest in the last few years has been in descriptive set theory, and in particular in the structure of the Wadge hierarchy — which in essence is the general theory of boldface pointclasses, i.e., collections of sets of reals closed under continuous preimages. The axiom of determinacy imposes a detailed structure on these pointclasses, but in fact it seems that most of the properties follow from a seemingly weaker principle, the so-called semi-linear ordering principle (SLO): for any pair of sets of reals A and B,

$$A \leq_{\mathrm{W}} B \ \vee \ \neg B \leq_{\mathrm{W}} A$$

where $\leq_{\mathrm{W}}$ is the relation of continuous preimage. Solovay has conjectured that SLO is equivalent to AD, at least if $V = L(\mathbb{R})$. If instead of considering *continuous* preimages we use other kind of functions (e.g.: Borel) we obtain a coarser comparability relation, which are of interest in their own right. A curious phenomenon that occurs under AD is that there are more $\boldsymbol{\Sigma}^0_\beta$ than $\boldsymbol{\Sigma}^0_\alpha$ when $\alpha < \beta$. The detailed analysis of the Wadge hierarchy has been used recently to pin-down the exact places at which a similar phenomenon occurs, by characterizing the pointclasses $\boldsymbol{\Gamma}$ which have a cardinality larger than any $\boldsymbol{\Lambda}$ contained in $\boldsymbol{\Gamma}$. Finally, I should point out that the Wadge hierarchy, besides being interesting per se, is important for constructing models of AD.

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My research has concentrated on weakly compact cardinals and the ideal naturally associated with them. A subset E of a regular cardinal κ is called Π^1_1-*indescribable* or *weakly compact* if for every Π^1_1-sentence ϕ and every $U \subseteq \kappa$ such that $\langle V_\kappa, \in, U \models \phi\rangle$ there exists an ordinal $\alpha \in E$ such that $\langle V_\alpha, \in, U \cap \alpha\rangle \models \phi$. Thus κ is a weakly compact cardinal iff there exists a weakly compact subset of κ or equivalently if κ is weakly compact as a subset of itself.

The *weakly compact ideal* consists of those subsets of κ that are not weakly compact. It is a normal ideal. It seems that weak compactness of sets is a remarkably natural generalisation of stationarity. This is one of the motivational factors behind the research. The phenomenon is also easily seen to generalise to the ideals associated with Π^1_n-indescribable cardinals for $n < \omega$.

Subtlety and ineffability, various diamond principles and saturation properties of the ideals are investigated. The weakly compact diamond is just like the ordinary diamond, except that guessing happens on a weakly compact set rather than just a stationary set. Weakly compact diamond holds on an ineffable cardinal whereas the classical diamond restricted to the regulars is known to hold on a subtle cardinal. One question we try to look into is whether κ can be subtle even though the weakly compact diamond fails. Other "small large cardinals" such as strongly unfoldables are of interest too.

It is consistent relative to a measurable cardinal that the weakly compact ideal over κ is not κ^+-saturated. How this result can be generalised to Π^1_n even for $n = 2$ is still unsolved.

The weakly compact ideal over κ is nowhere κ-saturated. It is open whether this result holds for ordinal (or weak) Π^1_1-indescribability, the concept that arises when inaccessibility is dropped from weak compactness. Characterisations via elementary embeddings and Π^1_1-sentences work in this kind of setting too, but some of the known characterisations of weak compactness imply inaccessibility. Models with large cardinals but many weak inaccessibles that are not inaccessible seem to be rather obscure, but motivation to pursue research in this direction can come from the thought that the true combinatorics underlying weak compactness can be better understood if inaccessibility is not in the picture.

All forcing arguments used so far have involved iterations with Easton supports (reverse Easton). It seems difficult or even impossible to find a really useful general preservation theorem for weak compactness. An observation on a very intuitive level is that elementary embeddings tend to be useful in forcing arguments wheras they do not seem to work very well in arguments that stay in one particular model of set theory. New ideas may be needed for results such as the nowhere κ-saturation mentioned above.

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I am looking at the inter-relations between various combinatorial principles, particularly the problem of whether $\clubsuit \rightarrow \exists x(x \text{ is a Suslin tree})$; and what can be proved regarding strengthenings or weakenings of these statements using forcing.

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Models of topological set theory. In Zermelo-Fraenkel set theory the contradictory naive comprehension scheme is weakened to the separation scheme, that is, it is restricted to formulas of the form $x{\in}a \wedge \varphi(x)$. In positive set theories one restricts the comprehension scheme to another class of formulas, namely the generalized positive ones, which are defined recursively just like formulas in general, but with the negation step weakened to bounded universal quantification. Since intersections, binary unions and the universe $\mathbb{V}$ are all defined by generalized positive formulas, this axiom scheme implies (given that the empty set exists) that the universe is a topology on itself. Moreover, it turns out to be finer than or equal to its own exponential topology. Conversely, the topological set theory **TopS** consists of topological axioms from which the generalized positive comprehension scheme follows. Its objects are classes, for which extensionality, full comprehension and a global choice principle hold. For the class of all sets $\mathbb{V}$, the following purely topological axioms are postulated:

- $\mathbb{V}$ is a κ-compact κ-topology on itself in the sense of closed sets, where κ=On.
- $\mathbb{V}$ is its own exponential topology.

Since in a κ-compact κ-topology the sets of size less than κ are exactly the discrete ones, the first axiom can be formulated in terms of discreteness without referring to the concept of an ordinal number. As an axiom of infinity, one can demand that the class of natural numbers ω be a set.

In my diploma thesis I showed how the usual set theoretic constructions can be carried out in **TopS**, proved some regularity properties of the universe's topology and roughly estimated its consistency strength by giving two natural models in **ZFC** with a weakly-compact cardinal, and in **TopS** a model of Kelley-Morse set theory in which the class of ordinals is weakly-compact.

I now intend to further determine its consistency strength and hopefully even find a variant of Kelley-Morse set theory which is mutually interpretable with **TopS**. I also want to investigate variants of the two models or new models to find out about the dependencies of some additional axioms like:

- The universe's topology is induced by a natural ultrametric.
- There exists a (class-)well-order on the universe.
- The well-founded sets are dense in $\mathbb{V}$. (Foundation axiom)
- To every [finite/discrete] extensional structure $\langle A,e\rangle$ there exists a transitive set X such that $\langle X,\in\rangle$ is isomorphic to $\langle A,e\rangle$. (Antifoundation axiom)

Furthermore, in the context of positive set theories and exponential topologies it is natural to weaken the topological axioms in such a way that they don't imply the existence of the empty set. Such a theory might – unlike **TopS** – have models without isolated points. One might also admit urelements and see whether these weaker topological set theories have a lower consistency strength.

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My research centers around large cardinal axioms, class forcing, and combinatorial properties that hold in L. In my doctoral thesis (working with Sy Friedman), I give a class forcing which endows the extension universe with a definable well-order, while preserving proper classes of "local" large cardinals. By this I mean large cardinals axioms that are witnessed by boundedly many elementary embeddings. I obtain the well-order by coding information into whether the principle $\diamondsuit^*_\kappa$ (a strengthening of $\diamondsuit_\kappa$) holds for various successor cardinals κ.

In my thesis I also show that one may force gap-1 morasses to exist at every uncountable regular cardinal while preserving all n-superstrong ($1 \leq n \leq \omega$), hyperstrong, and 1-extendible cardinals. Morasses are combinatorial structures found in L in which every element of some cardinal κ^+ is built up in a very structured way through the branches of a tree of size κ. Alongside that result, I give a forcing that yields universal morasses on regular cardinals κ — these are a kind of morass on κ that carry with them an encoding of $\mathcal{P}(\kappa)$. One may preserve a given n-superstrong, hyperstrong or 1-extendible cardinal while carrying out this latter forcing.

I also have a strong interest in the application of set theory to algebraic topology. Specifically, it was recently shown by Casacuberta, Scevenels and Smith that assuming Vopěnka's Principle (a large cardinal axiom above supercompact but below huge), one can settle (in the affirmative) the long standing question of whether all generalised cohomology theories have localisation functors. However, I do not have any results in this area as yet.

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My primary research interest is *singular cardinals combinatorics*.

Background. Singular cardinals happens to have a non-trivial effect on usual mathematical objects. From one hand, being limit cardinals, the singular cardinals satisfy some plausible compactness properties, e.g., Shelah's theorem that every group of singular cardinality in which every subgroup of smaller cardinality is free - is itself free. On the other hand, being the limit of less than itself many smaller cardinals, it is often possible to carry out diagonalization arguments with respect to these cardinals, establishing implausible properties, e.g., Pouzet's theorem that every poset whose cofinality is a singular cardinal must contain an infinite antichain.

The most interesting case is whenever the question of satisfaction of a certain property of a singular cardinal is determined by its cardinal arithmetic configuration, e.g., as in [5]. The research of singular cardinals combinatorics centers at determining the exact interplay between different cardinal arithmetic configurations and related combinatorial properties.

Work so far. In their paper from 1981, Milner and Sauer conjectured that the following improvement of Pouzet's above-mentioned theorem should hold: if $\langle P, \leq \rangle$ is a poset whose cofinality is a singular cardinal λ, then P must contain an antichain of size $cf(\lambda)$. Shortly afterwards, it has been observed by several authors including Hajnal, Prikry, Pouzet, and also Milner and Sauer that the conjecture is a consequence of GCH and some of its variants. However, to these days, a consistent counterexample is still unknown to exist.

In the last few years, a progress on this matter has been made in the form of an unpublished result of Magidor and independently the main result of [1], establishing that the Milner-Sauer conjecture has large cardinals consistency strength. Then, in [3], by pushing further the combinatorics of [2], it has been proved that this conjecture is a consequence of a certain, rather sharp, weakening of the GCH, whose consistency of its negation is a major open problem of modern cardinal arithmetic:

Definition (*Prevalent Singular Cardinals Hypothesis*)**.** *For every singular cardinal λ, there exists a family $\mathcal{A} \subseteq \mathcal{P}(\lambda)$ of size λ with* $\sup\{|A| \mid A \in \mathcal{A}\} < \lambda$ *such that every $B \subseteq \lambda$ of size $< cf(\lambda)$ is contained in some $A \in \mathcal{A}$.*

To describe another problem of a similar flavor, recall that given a topological space $\mathbb{X}$, its *density* describes the minimal (infinite) cardinality of a dense subspace, and its *weight* describes the minimal cardinality of a base for $\mathbb{X}$. By a recent exciting result of Juhász and Shelah, the existence of a regular hereditarily Lindelöf space of density $\aleph_{\omega_1}$ is consistent. Now, a diagonalization argument of [1] shows that such space must have more than $\aleph_{\omega_1}$ many open sets, but what about its weight? can it equal the density while preserving the hereditarily Lindelöfness?

In [4], it is proved that the existence of an hereditarily Lindelöf space of density and weight $\aleph_{\omega_1}$ entails the negation of the prevalent singular cardinals hypothesis.

Future plans. In order to determine whether the cardinal arithmetic configuration that was found to witness the negation of the Milner-Sauer conjecture (or the topological problem) indeed suffices to produce a counterexample, it is needed to find several ways to utilize negative cardinal arithmetic configurations in combinatorial constructions. An humble step in this direction appears in [5]. We shall also study Prikry-type forcing, specifically Gitik's forcing along short extenders, aiming at establishing the consistency of the cardinal arithmetic occurring in our combinatorial research, and trying to answer other well-known questions in the field of cardinal arithmetic and singular cardinals combinatorics.

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My research area is forcing technics of adding new reals related to special MAD families and cardinal invariants of the continuum. My advisor, Lajos Soukup and I are working on the following paper right now:

Invariants of analytic P-ideals and related forcing problems. (only a possible title) An ideal $\mathcal{I}$ on ω is *analytic* if as a subset of the space $\mathcal{P}(\omega)$ with the usual topology (i.e. Cantor-set) is analytic; $\mathcal{I}$ is a *P-ideal* if for each countable $\{I_n : n \in \omega\} \subseteq \mathcal{I}$ there is an $I \in \mathcal{I}$ such that $I_n \subseteq^* I$ (i.e. $|I_n \setminus I| < \omega$) for each n. It is well-known that each analytic P-ideal is of the form $\mathrm{Exh}(\varphi) = \{X \subseteq \omega : \lim \varphi(X \setminus n) = 0\}$ where $\varphi : \mathcal{P}(\omega) \to [0, \infty)$ is a *finite lower semicontinuous submeasure*. The main examples of such ideals are *density* and *summable* ideals.

Density ideals: Let $\{P_k : k \in \omega\}$ be a partition of ω into pairwise disjoint finite sets and let $\vec{\mu}$ be a sequence $\langle \mu_k : k \in \omega \rangle$ of measures such that μ_k is concentrated on P_k and $\limsup \mu_k(P_k) > 0$. Let $\mathcal{Z}_{\vec{\mu}}$ be the following ideal on ω:

$$\mathcal{Z}_{\vec{\mu}} = \{X \subseteq \omega : \lim \mu_k(X \cap P_k) = 0\}.$$

Ideals of this form are called density ideals. The ideal of asymptotic density zero sets, $\mathcal{Z} = \{A \subseteq \omega : \lim \frac{|A \cap n|}{n} = 0\}$ is a density ideal.

Summable ideals: Let $h : \omega \to \mathbb{R}^+$ be a function with $\sum_{n\in\omega} h(n) = \infty$ and let $\mathcal{I}_h$ be the following ideal on ω:

$$\mathcal{I}_h = \Big\{A \subseteq \omega : \sum_{n\in A} h(n) < \omega\Big\}.$$

These ideals are called summable ideals. For example the ideal of finite sets is a summable ideal.

Let $\mathcal{I}$ be an ideal on ω, and let $\mathcal{I}^+ = \mathcal{P}(\omega)\setminus\mathcal{I}$. An infinite family $\mathcal{M} \subseteq \mathcal{I}^+$ is *$\mathcal{I}$-almost disjoint ($\mathcal{I}$-AD)* if $A \cap B \in \mathcal{I}$ for each distinct $A, B \in \mathcal{M}$. An $\mathcal{I}$-AD family $\mathcal{M}$ is *maximal ($\mathcal{I}$-MAD)* if for each $X \in \mathcal{I}^+$ there is an $A \in \mathcal{M}$ such that $X \cap A \in \mathcal{I}^+$, that is, $\mathcal{M}$ is $\subseteq$-maximal among $\mathcal{I}$-AD families. The *almost disjoint number of $\mathcal{I}$*, denoted by $\mathfrak{a}_\mathcal{I}$ ($\mathfrak{a}^*_\mathcal{I}$), is the minimum of the cardinalities of (uncountable) $\mathcal{I}$-MAD families. We have proved the following results:

$\mathfrak{a}_{\mathcal{I}_h} > \omega$ for each summable ideal $\mathcal{I}_h$ and $\mathfrak{a}_{\mathcal{Z}_{\vec{\mu}}} = \omega$ for most density ideals.

$\mathfrak{a}^*_{\mathcal{Z}_{\vec{\mu}}} \leq \mathfrak{a}$ for each density ideal $\mathcal{Z}_{\vec{\mu}}$, where $\mathfrak{a} = \mathfrak{a}_{[\omega]^{<\omega}}$ is the well-known almost disjointness number.

$\mathfrak{b} \leq \mathfrak{a}^*_\mathcal{I}$ for each analytic P-ideal, where $\mathfrak{b}$ is the unbounding number of $\langle \omega^\omega, \leq^* \rangle$.

We are working on related forcing questions as well. Let V be a transitive model of (a large enough segment of) ZFC. An $X \subseteq \omega$ is a *$\mathcal{Z}$-covering real over V* if $X \in \mathcal{Z}$ and $A \subseteq^* X$ for each $A \in \mathcal{Z} \cap V$. Results:

If $V \subseteq W$ are models and W contains a $\mathcal{Z}$-covering real over V then W contains a dominating real over V as well.

If $V \subseteq W$ are models and W contains a slalom over V, that is, an $S \in W$, $S : \omega \to [\omega]^{<\omega}$, $|S(n)| \leq n$, and for each $f \in \omega^\omega \cap V$ $f(n) \in S(n)$ for all but finite n, then W contains a $\mathcal{Z}$-covering real over V. Specially the Localization-forcing ($\mathbb{LOC}$) adds $\mathcal{Z}$-covering reals.

There is a natural ccc forcing too which adds a $\mathcal{Z}$-covering real over the ground model but a σ-centered forcing notion cannot add such a real.

A forcing notion $\mathbb{P}$ has the Sacks-property if, and only if $\mathbb{P}$ is *$\mathcal{Z}$-bounding*, that is, $\mathbb{P}$ forces that for each new element of $\mathcal{Z}$ can be covered by an element of $\mathcal{Z}$ from the ground model.

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My general area of interest is hypercompution. Within this area I study ordinal time Turing computation. OTTM's admit several reducibility notions. I am interested in studying these and comparing them to established infinitrary reducibilities (in particular α-recursion).

Ordinal time Turing computation also has strong links with constructibility. I am looking at the constructible universe afresh with the aid of these machines and the induced reducibility notions.

There is also a substantial question as to what languages are accepted by OTTMs. Generalisations of the lost melody theorem become significant here. I conjecture that sharps are lost melodies for the OTTMs.

In addition to ordinal time Turing computation I have formulated some slight extensions of OTTMs which are capable of computing sharps and various mice. Introducing a notion of strong computability I am interested in what axioms the universe of strongly computable sets is closed under (starting with replacement and power set).

I am investigating these extensions further with a view to proving that they allow the computation of inner models of measurability and possibly other inner models of large cardinals. I would like to find a characterisation of those sets computatable by these extensions. I would also like to characterise which axioms that hold in V will reflect down (in a strong sense) to the universe of strongly computable sets.

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I mainly work in infinitary combinatorics, in particular

- the combinatorial structure of the universe and of inner models
- the construction of forcing notions using combinatorial principles
- applications, e.g. in topology.

In my Ph.D. thesis I gave an analysis of the combinatorial structure of inner models L[X] where X satisfies codensation, amenability and coherence. For this, I generalized Jensen's notion of higher-gap morasses to gaps of arbitrary size and proved that they in a certain sense completely capture the combinatorial structure of such models.

My present work is concerned with the construction of forcing notions along morasses. The idea is simple: I generalize the classical notion of iterated forcing with finite support. However, instead of considering linear systems of forcings and taking direct limits, I consider higher-dimensional systems and take the morass limit. Like in the case of normal finite support iterations, chain conditions are preserved.

As first applications, I constructed (1) a ccc forcing of size ω_1 that adds an ω_2-Suslin tree, (2) a ccc forcing that adds a chain $\langle X_\alpha \mid \alpha < \omega_2 \rangle$ such that $X_\alpha \subseteq \omega_1$, $X_\beta - X_\alpha$ is finite and $X_\alpha - X_\beta$ has size ω_1 for all $\alpha < \beta < \omega_2$ and (3) a ccc forcing of size ω_1 that adds a 0-dimensional T_2 topology on ω_3 which has spread ω_1.

Applications (1) and (2) use two-dimensional systems, i.e. gap-1 morasses, while application (3) uses a three-dimensional system, i.e. a gap-2 morass. That (2) holds, was first shown by Piotr Koszmider with Todorcevic's method of rho-functions. However, in (3) even the consistency of the existence of such a topology was open.

In preperation: It is consistent that there exists a function $g : [\omega_3]^2 \to \omega_1$ such that $\{\xi < \alpha \mid g(\xi, \alpha) = g(\xi, \beta)\}$ is finite for all $\alpha < \beta < \omega_3$.

Related results are contained in S. Todorcevic's book "Walks on ordinals and their characteristics".

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In my thesis, I introduce a two-player game $G(f)$ which "characterizes" the Borel functions on the Baire space, in the sense that Player II has a winning strategy in $G(f)$ if and only if f is Borel. In this game, there are two players who alternate moves for ω rounds. Player I plays natural numbers $x_i \in \omega$ and Player II plays functions $\phi_i : T_i \to {}^{<\omega}\omega$ such that $T_i \subset {}^{<\omega}\omega$ is a finite tree, ϕ_i is monotone and length-preserving, and $i < j \Rightarrow \phi_i \subseteq \phi_j$.

I:	x_0		x_1		x_2			$x = \langle x_0, x_1, \dots \rangle$
							$\dots$	
II:		ϕ_0		ϕ_1		ϕ_2		$\phi = \bigcup \phi_i$

After infinitely many rounds, Player I produces x and Player II produces ϕ as shown. Player II wins the game if and only if $\mathrm{dom}(\phi)$ has a unique infinite branch z and

$$\bigcup_{s \subset z} \phi(s) = f(x).$$

One of the results of my thesis is that Player II can guarantee victory in this game precisely when f is Borel. This is a generalization of the Wadge game, which characterizes the continuous functions in a similar way.

By adding extra rules for Player II, it is possible to characterize subclasses of Borel functions. In particular, it is possible to characterize Baire class 1 and Baire class 2. Using game-theoretic methods, I proved decomposition theorems for two subclasses of Baire class 2 (see notation section for what is meant by $n{\to}m$):

A function $f : {}^\omega\omega \to {}^\omega\omega$ is $2{\to}3 \Leftrightarrow$ there is a $\mathbf{\Pi}^0_2$ partition $\langle A_n : n \in \omega\rangle$ of ${}^\omega\omega$ such that $f \restriction A_n$ is Baire class 1.

A function $f : {}^\omega\omega \to {}^\omega\omega$ is $3{\to}3 \Leftrightarrow$ there is a $\mathbf{\Pi}^0_2$ partition $\langle A_n : n \in \omega\rangle$ of ${}^\omega\omega$ such that $f \restriction A_n$ is continuous.

At least for the Baire space, this extends the result of Jayne and Rogers (1982) that f is $2{\to}2$ if and only if there is a closed partition A_n such that $f \restriction A_n$ is continuous.

Notation:
The symbol ω denotes the set of natural numbers,
${}^{<\omega}\omega$ is the set of finite sequences of natural numbers,
${}^\omega\omega$ is the set of infinite sequences of natural numbers,
$T \subseteq {}^{<\omega}\omega$ is a tree if $t \in T$ and $s \subset t \Rightarrow s \in T$,
$\phi : T \to {}^{<\omega}\omega$ is monotone if $s \subseteq t \Rightarrow \phi(s) \subseteq \phi(t)$,
ϕ is length-preserving if $\mathrm{lh}(\phi(s)) = \mathrm{lh}(s)$, and
f is $n{\to}m$ if $f^{-1}[X] \in \mathbf{\Sigma}^0_m$ for every $X \in \mathbf{\Sigma}^0_n$.

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Subtle and Ineffable Tree Properties. It is well known that an inaccessible κ is Mahlo iff there exists no special κ-Aronszajn tree and that it is weakly compact iff there exists no κ-Aronszajn tree (which we will abbreviate by κ-TP). For $(T, <_T)$ a tree[3] let us define the *subtle tree property* STP and the *ineffable tree property* ITP:

(STP(T)) If $\mathrm{ht}(T) = \kappa$, $C \subset \kappa$ club, $\langle t_\alpha \mid \alpha \in C\rangle \in \prod_{\alpha\in C} T_\alpha$, then there are $\alpha, \beta \in C$ such that $t_\alpha <_T t_\beta$,

(ITP(T)) If $\mathrm{ht}(T) = \kappa$, $\langle t_\alpha \mid \alpha < \kappa\rangle \in \prod_{\alpha<\kappa} T_\alpha$, then there is a stationary $S \subset \kappa$ such that $\{t_\alpha \mid \alpha \in S\}$ is a $<_T$-chain.

Now it is obvious from the usual definitions that an inaccessible κ is subtle iff every κ-tree T satisfies $\mathsf{STP}(T)$, for which we shall just write κ-STP, iff the complete binary tree $2^{<\kappa}$ satisfies $\mathsf{STP}(2^{<\kappa})$, and similarly for ineffability (and one can take this as a definition if unfamiliar with the concepts).

By [Mit73] one can collapse a weakly compact (a Mahlo) cardinal onto ω_2 such that in the resulting universe there exists no ω_2-Aronszajn tree (no

[3] We require all trees to not split at limit levels, i.e. if δ is limit and $s, t \in T_\delta$ are such that $\{u \in T \mid u <_T s\} = \{u \in T \mid u <_T t\}$, then $s = t$. Otherwise the following concepts would just trivially be wrong.

special ω_2-Aronszajn tree), and if there are no ω_2-Aronszajn trees (no special ω_2-Aronszajn trees), then (κ is weakly compact)^L ((κ is Mahlo)^L) holds. One can do the same for subtlety and ineffability, so that the existence of a subtle or an ineffable cardinal is also equiconsistent with the truth of certain combinatorial principles for ω_2.

In [MS96] it is shown that if λ is the singular limit of strongly compact cardinals, then λ^+-TP holds—what about λ^+-STP or λ^+-ITP? Furthermore the consistency of ω_ω^+-TP is proved under some large cardinal assumptions, so can we get ω_ω^+-STP or ω_ω^+-ITP here? Baumgartner showed PFA implies ω_2-TP (see [Tod84, chap. 7] or [Dev83, §5]), so we would like to know if PFA also implies ω_2-STP or ω_2-ITP.

We can further generalize these properties to get ideals similar to the approachability ideal. For example we can consider the ideal of all subsets B of κ such that some κ-tree has an antichain which has an element of height β for every $\beta \in B$, so that κ-STP becomes the property this is a proper ideal. One can also reduce the requirement of having a tree with an antichain to having an antichain where the initial segments are enumerated before, so that we get an ideal containing the approachability ideal. We are then led to the question if for example on ω_2 these ideals can be the nonstationary ideals on $\mathrm{cof}(\omega_1)$, cf. [Mit05].

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Forcing absoluteness and Regularity properties.

Backgrounds and results.

Forcing absoluteness is one of the central topics in set theory and it connects many areas in set theory while regularity properties are nice properties for sets of reals which have been deeply investigated for many years. There is a close connection between forcing absoluteness and regularity properties, e.g. all the Σ^1_3-formulas are absolute between V and its Cohen forcing extensions iff every $\mathbf{\Delta}^1_2$-set of reals has the Baire property iff for any real a, there is a Cohen real over $L[a]$. The same kind of equivalence holds for random forcing and Lebesgue measurability, Hechler forcing and the Baire property for dominating topology, and Mathias forcing and Ramsey property etc. In my master's thesis [3], I proved the equivalence for Sacks forcing and Bernstein property (a set of reals has the Bernstein property if it or the complement of it contains a perfect set). In this case, the statement for generic reals cannot be added to the equivalence, i.e. the statement "for any real a, there is a Sacks real over $L[a]$" is stronger than the other two statements. Instead, the statement "for any real a there is a real which is not in $L[a]$" is equivalent to them. Recently, I have succeeded to introduce a large class of forcing notions from each of which we can define the corresponding regularity property and to prove the equivalence in each case in a uniform way. This class contains all the practical forcing notions and this result implies the unknown equivalence for some forcings (e.g. Miller forcing and Silver forcing). Also this result solves one open question in the paper "Silver measurability and its relation to other regularity properties" by Brendle, Halbeisen and Löwe [2].

Future directions and questions

(1) Generic reals.

Although the existence of generic reals for L is too strong for the equivalence in the case of Sacks forcing as I mentioned above, we have a weaker notion of generic reals so called "quasi-generic reals" and for these reals, the equivalence for the three statements holds even in the case of Sacks forcing. The question is if we can generalize this relationship of three statements up to any forcing in the class I have introduced. The answer is true if we restrict our attention to ccc $\mathbf{\Sigma}^1_2$ forcings.

(2) Higher level forcing absoluteness and regularity properties.

The relation I have mentioned is only for Σ^1_3-forcing-absoluteness and regularity properties for $\mathbf{\Delta}^1_2$ sets of reals. The question is if we can generalize this relationship up to Σ^1_{n+1}-forcing-absoluteness and regularity properties for $\mathbf{\Delta}^1_n$ sets of reals. When $n = 3$, we have affirmative answers for Cohen forcing, random forcing and Sacks forcing although we need large cardinal assumptions. The goal of this question is to find the optimal assumption to prove the equivalence for each n.

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Recently I have been looking at the following type of problem:

(•) (In ZFC, without conditions on the universe) Find a set–forcing extension of the universe in which $H({}_2)$ admits a definition, over the structure $\langle H({}_2), \in \rangle$, which is simple in the sense of logical complexity and, more importantly, which does not use any parameters.

I have also worked on several variations of the above question. One such variation is to ask for a definable well–order of $H({}_2)$, always without parameters, together with some strong from of forcing axiom. Another one is to replace ${}_2$ with κ^+ for an any uncountable regular cardinal κ, and even to look for a locally definable well–order of the universe while preserving large cardinals.

In order to deal with these problems I have developed various (quite unrelated) coding techniques. The extensions are always built using iterated forcing constructions.

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A set of reals (in $\mathbb{R}^n$, or more generally in any Polish space) is projective if it arises from an open (or Borel-) set (in $\mathbb{R}^k$) through repeatedly taking the complement or the image under projection maps. An example of a regularity property would be that of being Lebesgue-measurable, or that of having the property of Baire (these being the two examples studied in my thesis).

More generally, given a σ-ideal I on $\mathbb{R}$, we call a set of reals I-regular if and only if it is equal to a Borel set, modulo a set from I. Numerous examples of such ideals are well-studied, as they naturally arise in forcing theory of the reals.

It has long been known that we cannot, in ZFC, settle the question whether projective sets are Lebesgue-measurable (or have some other regularity property of your choice).

Two things are surprising in this field of study. Firstly, it should be possible to force every "simple" set (that is, up to a given level of the projective hierarchy) to be regular (in some specified sense), with regularity failing at the next level. Nonetheless, this has not been done yet from an optimal assumption, that is, the proof uses too strong hypotheses in the sense of assuming the consistency of relatively large cardinals. Secondly, until recently, all the known techniques to tamper with the regularity of sets in the projective hierarchy affected all the notions of regularity simultaneously. Yet in the general case it should not be expected that every set up to a certain level of complexity having regularity property A should mean that all of these sets have to be regular in a different sense, B.

The techniques developed in my thesis open up several possibilities dealing with questions such as these. My thesis solves the following case: we have a model where every projective set of reals is Lebesgue-measurable but there is a set without the Baire property, at the lowest level possible in the projective hierarchy. It remains to be seen if we can generalize this to be able to prescribe at which level non-regular sets occur. And it remains to be seen if we can do similar constructions with other notions of regularity. Both questions will not be solved by straightforward generalisations of the proof mentioned. The second question has to do with finding properties of ideals which, in a sense, allow to distinguish between the reals added by certain forcing notions.

This question is also of relevance to the intricate theory of what reals are added by a forcing. In fact, there are reasons to expect this line of research to yield interesting examples of adding reals, contributing to the general theory of forcing.

Another topic in descriptive set theory to which the techniques developed in my thesis may prove useful is the theory of so-called "small sets". I plan to investigate possible applications in this area.

Further research should also address the question whether we can adapt the forcing used to prove the result in my thesis in such a way as to allow for models where the continuum hypothesis fails; if it is the case that we have found a general approach to dealing with the regularity of projective sets, it should not rely on forcing the continuum to be $\aleph_1$ (as does the construction given in my thesis).

There is also no apparent reason that we should be confined to starting with L as a ground model. Being able to carry out the same construction starting from models incorporating large cardinals and then showing that some of their largeness is preserved would be a big step toward a complete understanding of regularity properties of projective sets; provided we can take it up to the level of large cardinals where "they take over" and imply regularity (up to some level), thus ruling out the kind of freedom our models would exploit.

Lastly, an important question left open in my thesis is the optimality of the large cardinal hypothesis in the proof. Although the hypothesis we use is very mild (the existence of a Mahlo cardinal), one would like to prove that this is the weakest possible hypothesis; this is not at all apparent. One scenario would be to try develop the theory of forcing iteration used in my thesis further, to show the proof actually works with a weaker assumption. But the other scenario - that the existence of a Mahlo is in fact optimal - is also feasible.

The general theory of iterated forcing is another field I plan to do research in. One of the questions I intend to address has to do with preservation theorems. A preservation theorem tells us that an iteration of forcing does not do unwanted damage if none of the iterands does. The known preservation theorems usually use small supports in the iteration, a harsh technical limitation. My thesis has an interesting example of an iteration which does no harm, yet none of the known iteration theorems apply. Moreover, this iteration uses an interesting kind of support. I plan to work on the question of finding a "nice" class of forcing, closed under iteration, which should include the forcing used in my thesis.

Another respect in which my future research can build on the iteration theory developed in my thesis is the question of which large cardinals are preserved in a forcing extension. In my thesis, I describe a class of forcing, dubbed stratified. This notion should be applicable when trying to retain large cardinal properties (e.g. measurability) when going to a forcing extension.

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Inner Model Theory I compared the fine structures that arise in two different approaches to constructing extender models (definable submodels of a universe of set theory which have a canoncial structure while reflecting strong axioms of infinity true in the universe), as pursued by William Mitchell and John Steel on the one hand, and Sy Friedman and Ronald Jensen on the other. I established a one-to-one correspondence between the two models' building blocks. I developed a method of translating formulae between the corresponding structures. It also translates iteration strategies for these structures.

In my post-doctoral work, I pursued the problem of iterability, i.e., of the existence of iteration strategies, further, as it is central to inner model theory.

Forcing *Příkrý forcing:* I introduced a generalization and characterized the corresponding generic sequences combinatorially, connecting them to inner model theory both ways.

Forcing axioms: The maximality principle for $<\kappa$-closed forcings, for a regular cardinal κ, says that any statement that can be forced by a $<\kappa$-closed forcing to be true in such a way that it stays true in any further forcing extension of that kind, is already true. I investigated these principles and will look at their connections to modal logic. They have many interesting consequences, and can be combined. But the strongest thinkable combinations of these principles is inconsistent. Can they hold at every regular cardinal below some cardinal satisfying some strong axiom of infinity? This is a topic that I am currently investigating.

Set Theoretic Geology: Together with Joel Hamkins and Jonas Reitz. The starting point is the result of Richard Laver, which was obtained independently by Hugh Woodin, that the ground model is definable in each of its forcing extensions. We are now trying to define and investigate canonical inner models of a given model, using this method of "inverting forcing".

Interactions between Algebra and Set Theory The automorphism tower of a centerless group: A centerless group embeds into its automorphism group, which is again a centerless group. Iterating this process leads to a sequence of centerless groups, each embedded into the next. At limits, it is possible to form the direct limit; one gets yet another centerless group, and so the process can be continued transfinitely. The groups occurring in this sequence form the automorphism tower of the original group. The first index of a group in the automorphism tower that is isomorphic to the next group is called the height of the automorphism tower. In work on rigidity degrees of Souslin trees, Joel Hamkins and I developed methods for subtly coding automorphisms of one tree into branches of another tree. Building on this, we were able to prove that in L, there are groups whose automorphism tower heights are highly malleable by forcing: The height of the automorphism tower of the very same group can be very different in different forcing extensions of L.

Large Cardinals Combined closed maximality principles up to a large cardinal: I use "Woodinized" versions of known large cardinal concepts - these are ideas due to Matthew Foreman, and techniques of lifting elementary embeddings to generic extensions, mostly of the kind introduced by Silver.

Indestructible Weak Compactness: In the aforementioned area I came across the concept of an indestructible weakly compact cardinal quite naturally: If the closed maximality principle holds on a measure one set below a measurable cardinal, then there is a measure one set of indestructible weakly compact cardinals below this measurable cardinal. Indestructible weak compactness gives rise to generic embeddings in a natural way, which can be used to prove results on the stationary tower and on strong forcing axioms like $\mathsf{MA}^+(\sigma - \text{closed})$.

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I am working on cardinal characteristics, combinatorics in $\aleph_1$, and sometimes I work with large cardinals.

The main technique in my work is forcing, and an important aim on the technical side is to develop forcing techniques. I also have been working in the analysis of existing notions of forcing with respect to new properties. Cardinal characteristics of the continuum often depict important combinatorial features of the ZFC models in question. A cardinal characteristic of the continuum locates the smallest size of a set with a property that is usually not exhibited by any countable set and that is exhibited by at least one set of size of the continuum. However, sometimes a mathematical statement is derived from some delicate stratification of the set-theoretic universe that cannot (yet) be reduced to cardinal equations. This can in particular be the case for set-theoretic universes that are established with not so conventional forcing constructions.

In the Bonn conference I will talk about my current work on weak diamonds, which is a continuation of [2] and [1]. We show that some weakenings of the club principle do not imply the existence of a Souslin tree. We show that $\diamondsuit(2^\omega, [\omega]^\omega,$
is constant on) together with CH and "all Aronszajn trees are special" is consistent relative to ZFC. This implies the analogous result for a double weakening of the club principle. In [2] we showed: There are completeness systems $\mathbb{D}$ such that proper $\mathbb{D}$-complete forcings can be so mild that a generic condition over a countable elementary submodel is even given by a Borel function of the code of the countable model. Now this is used in a modified way.

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My PhD project is officially about partition properties *without the axiom of choice* (AC), but I like all large cardinals in ZF. At the moment I study higher Chang conjectures. Their consistency strength seems to drop considerably without AC. For example look at $(\omega_4, \omega_2) \twoheadrightarrow (\omega_2, \omega_1)$ which is inconsistent with choice. This Chang conjecture and all the others that 'start' with a successor of a regular cardinal, are not only consistent with ZF but equiconsistent with only one Erdős cardinal. In ZFC other higher Chang conjectures of that form have high consistency strength. For example, Donder and Koepke showed that if $(\kappa^{++}, \kappa^{+}) \twoheadrightarrow (\kappa^{+}, \kappa)$ and $\kappa \geq \omega_1$ then $0^{\dagger}$ exists. At the moment I'm working on a presentation of the consistency strengths of the Chang conjectures in ZF, as complete as possible.

Other large cardinals that become weaker or "small" without choice, are ones whose definition involves partitions and/or ultrafilters. Jech showed that ω_1 can be measurable and it's easy to show that it can be also weakly compact, Ramsey and more. This indicates that having a normal measure is not a good definition for measurability and other large cardinals if you're in a choiceless world. There, elementary embeddings should be used.

Measurable cardinals are indeed very interesting to me. Even the simple question of whether one can have a measurable with no normal measures doesn't seem to have an obvious answer. I'm also puzzled with the problem of successive measurables; either having two or three in a row with assumptions below AD or having more than three successive measurables with *any* assumptions. This is a long standing open question. It apparently should be connected to Radin forcing and perhaps to Moti Gitik's construction where every uncountable cardinal is singular. Measurable cardinals and these methods for working with them are things I'd like to get to know much better.

Equiconsistency proofs in the large cardinal realm are usually done with forcing for one side and core models for the other. In the forcing side I like using symmetric forcing which produces models without choice. I use Jech's uniform method for symmetric forcing, only translated from Boolean valued models to forcing with partial orders. In the core model side, I have spent some time reading about the Dodd-Jensen core model. However, I'm happy to understand the basic ideas and just use black boxes from this very complicated theory. I do admire core model theory proofs and constructions but I prefer spending my time forcing. Therefore, core model theorists would be my main target for future collaboration.

Finally, another project I am working on with Peter Koepke is a paper on topological regularities in second order arithmetic (SOA). This is a project Peter Koepke had with Michael Möllerfeld. It is shown that ZFC is equiconsistent with SOA + "all sets of reals are Lebesgue measurable, have the Baire property and the perfect set property". I helped this project in the forcing side, a class Lévy collapse of all the ordinals. This project will be continued by studying this model for further topological regularities. Working on this has got me interested in class forcing, a very powerful method indeed.

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My PhD Project is about the *Mutual Stationarity Property* as defined by Matt Foreman and Menachem Magidor. The mutual stationarity property can be used to transport some properties of regular cardinals to singular limits of sequences of regular cardinals. I obtained a couple of results, partielly in joint work with Arthur Apter and/or Peter Koepke, on the mutual stationarity property; most of them are equiconsistency results on mutual stationarity in models where the axiom of choice fails. These results will be published in my forthcoming PhD-Thesis and in two publications.

The following techniques are the main ingredients in my research: *theory of partition cardinals and indiscernibles, forcing and symmetric model constructions, core model theory.*

In relation with this project I would be very happy if I could discuss some of the following issues at the Workshop:

(1) indiscernibles for measurable cardinals of higher Mitchell order,
(2) Radin forcing,
(3) Laver preparation and related techniques,
(4) the link between mutual stationarity, tight stationarity, and PCF-theory,
(5) core model theory,
(6) applications of core models in equiconsistency proofs.

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My research in set theory is focussed on consistency results, forcing, and combinatorics. I am especially interested in combinatorial properties related to stationary sets, for example, saturated ideals, the approachability property, and internally approachable models. In forcing I specialize in proper forcing style methods, iterated forcing, and methods for extending elementary embedding. I give two examples of open problems I am interested in.

The first problem concerns consistency results for saturated ideals. A famous theorem of set theory is the consistency of the statement that the non-stationary ideal on ω_1 is saturated. This statement means that there does not exist a collection of stationary subsets of ω_1 with size $\aleph_2$, which is an *antichain* in the sense that the intersection of any two sets in the collection is non-stationary. A natural problem is to generalize this consistency result to cardinals larger than ω_1. At a first glance, this appears to be impossible. For example, Shelah has proven that the non-stationary on ω_2 cannot be saturated, because any stationary subset of the set $\{\alpha < \omega_2 : \mathrm{cf}(\alpha) = \omega\}$ can be split into $\aleph_3$ many stationary subsets, any two of which have non-stationary intersection. However, it may be that the statement "the non-stationary ideal on ω_2 is saturated" is the wrong generalization of the saturation of the non-stationary ideal on ω_1. Since the limit ordinals below ω_1 all have cofinality ω, the non-stationary ideal on ω_1 is the *same ideal* as the non-stationary ideal on ω_1 restricted to ordinals with cofinality ω. Perhaps then the correct generalization is the statement that the non-stationary ideal on ω_2 restricted to cofinality ω_1 is saturated. Whether this statement about ω_2 is consistent is a well-known open problem in set theory.

The second problem I discuss involves singular cardinal combinatorics and forcing. Define the *approachability ideal* $I[\aleph_{\omega+1}]$ as the collection of sets $A \subseteq \aleph_{\omega+1}$ such that there exists a sequence $\langle a_i : i < \aleph_{\omega+1}\rangle$ of bounded subsets of $\aleph_{\omega+1}$ such that for club many α in A, there is an unbounded set $c \subseteq \alpha$ with order type equal to $\mathrm{cf}(\alpha)$ such that every initial segment of c is equal to a_i for some $i < \alpha$. An important theorem in singular cardinal combinatorics is Shelah's result that for all $n < \omega$, $I[\aleph_{\omega+1}]$ contains a stationary subset of $\aleph_{\omega+1} \cap \mathrm{cof}(\omega_n)$. If $\square_{\aleph_\omega}$ holds, then every subset of $\aleph_{\omega+1}$ is in $I[\aleph_{\omega+1}]$. On the other hand, Magidor proved that under Martin's Maximum, there is a stationary subset of $\aleph_{\omega+1} \cap \mathrm{cof}(\omega_1)$ which is not in $I[\aleph_{\omega+1}]$. A current open problem is whether $\aleph_{\omega+1} \cap \mathrm{cof}(> \omega_1)$ is in $I[\aleph_{\omega+1}]$. Personally I believe this is most likely false in general. But constructing a model in which, for example, there is a stationary subset of $\aleph_{\omega+1} \cap \mathrm{cof}(\omega_2)$ which is not in $I[\aleph_{\omega+1}]$ turns out to be quite difficult.

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Set theory and model-theoretic logics. One of the roles of logic is to serve as a tool for the study of structures. This role materializes in first order logic in the most spectacular way, as shown for example by the so-called Main Gap Theorem of Shelah. Typical of first order logic is that it cannot distinguish between cardinalities of infinite models. By Lindström's Theorem this property actually characterizes first order logic. There are many extensions of first order logic where sharper distinctions are possible. The most notable ones are the infinitary logics, logics with generalized quantifiers, and higher order logics. Infinitary logics are related to generalized recursion theory and set theory. Generalized quantifiers are related to model theory. Higher order logics are related to what could be called definability theory (following J. Addison).

There are also intermediate logics which do not fit well into the above three categories. A good example is the equicardinality quantifier "there are as many x with $\phi(x)$ as there are y with $\psi(y)$", which is a generalized quantifier and should therefore belong to the model theory category, but is actually an amalgam of model theory, set theory and definability theory. It is an example of a *strong* logic, that is, a logic which has enough power to express properties of not only this or that model, but of the underlying set theoretical universe. Other examples of strong logics are the Henkin quantifier, most higher order logics, and infinite quantifier logics $L_{\kappa\lambda}$, $\lambda > \omega$. The opposite is an *absolute* logic, that is, a logic the truth definition of which makes no reference to what kind of set there exists in the underlying universe. Typical examples of absolute logics are the infinitary logics $L_{\kappa\omega}$, enhanced perhaps with the game quantifier.

The first and foremost model theoretic properties of first order logic are the compactness property, the Löwenheim-Skolem property, the Craig interpolation property, and the axiomatization property. Each property has a life among extensions of first order logic, but often in a weaker form. By and large one expects that the logics in the "model theory" category, i.e. logics with generalized quantifiers, would permit such model theoretic properties as we just listed. It has turned out surprisingly difficult to prove such results. The truth is, the range of model theoretic properties extends all across the spectrum of logics, but in many cases only under subtle set theoretic assumptions.

In my work I try to get a clear picture of the set theoretic conditions on which model theoretic properties of extensions of first order logic depend. Note that the proofs of such properties do not always follow the line of argument familiar from first order logic. Genuinely new techniques have to be developed. Sometimes these techniques are or become standard tools in the model theory of first order logic (e.g. back-and-forth techniques, Vaughtian pairs). Sometimes they lead to set theoretic investigations completely detached from model theory (e.g. large cardinals, reflection principles for the continuum, the study of the ordering of trees).

In my recent work with Magidor we study a strong form of the Löwenheim-Skolem-Tarski property for the equicardinality quantifier and its relatives. To show that this property has great proof theoretic strength we prove that it implies failures of weak square and also that it implies the SCH. On the other hand we show, starting from a supercompact cardinal, that it can hold as low as on the first weakly inaccessible cardinal.

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Arithmetic and Reduced Powers The failure of categoricity for models of arithmetic, in that this stands to refute the idea that we can make our intuitions about arithmetic precise, is a fact of fundamental philosophical importance. One can still try to classify and/or describe these models, but non-standard models of arithmetic are not recursive, meaning that the set of triples belonging to the addition relation (or respectively the multiplication relation) of a (countable) non-standard model is not a recursive set.

Stanley Tennenbaum, who proved their non-recursiveness, also showed that any countable model of arithmetic is embeddable into the reduced power $\mathbb{N}^\omega/F$. The classification project then reduces to that of describing the behavior of (equivalence classes of) functions from $\mathbb{N}$ to $\mathbb{N}$ which happen to belong to models of arithmetic inside that structure. I obtained some results along these lines.

Subsequent research in collaboration with Jouko Väänänen and Saharon Shelah devolved on the question of whether uncountable models of arithmetic were embeddable into $\mathbb{N}^\omega/F$, also the same substituting any regular filter in place of the Frechet filter F, and more generally still the question whether for any first order structure M of any cardinality, every model elementarily equivalent to the reduced power M^λ/D and of cardinality $\leq \lambda^+$ is embeddable into M^λ/D, for D a regular filter.

Over a series of papers, the authors showed that the question as conjectured by Chang and Keisler is independent of ZFC, as is the related question whether $A^\lambda/D \cong B^\lambda/D$ whenever A and B are elementarily equivalent models of size $\leq \lambda^+$ in a language $\leq \lambda$ and D is a regular filter. This was done by isolating a principle equivalent to the original conjectures, namely a finitary square principle $\square^{fin}_{\lambda,D}$, a variant of $\square_\lambda$. Questions to be taken up in subsequent research are whether proving estimates for the consistency strength of $\neg\square^{fin}_{\lambda,D}$ can be obtained, and whether $\square^{fin}_{\lambda,D}$ has a similar relation as other square principles to axioms like PFA.

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As part of the inner model programme of S. Friedman, I am involved in finding internal consistency results for global properties. The properties that we consider are generalisations of cardinal characteristics such as the dominating and splitting numbers.

The bulk of my work is to classify certain relational structures using their combinatorial properties or through the universality programme, which heavily relies on combinatorial methods. I concentrate on three sub-projects to this line of research.

The first is to determine and find connections between universality spectra for non-elementary relational structures. A *universal model* at cardinality

κ is one which embeds all other structures in the set of those members of the class which have size κ, where an *embedding* is an injective structure-preserving map. There is a strong programme in universality and much progress has been made on using this indicator to classify elementary structures in a model-theoretic way. However, non-elementary structures are not model-theoretically well-behaved and so we rely on set-theoretic methods (in particular, forcing and combinatorics) to decide these questions.

The second is to classify orders (linear and partial) which have generalised notions of dense and scattered. One may define a notion of *κ-dense* (for some infinite cardinal κ) to be such that in between every two points, there is a set of size κ or there is a stronger notion where in between every two sets of size $< \kappa$ there is a point (equivalently κ many). Then *κ-scattered* may be defined for both notions as the property of not embedding a κ-dense set. These classifications take the form of a constructive hierarchy. For scattered orders, these constructive hierarchies proved to be a very useful tool for proving structure and combinatorial theorems about such orders and I plan to extend these results to orders which are κ-scattered (in either sense).

The third is to find purely combinatorial classifications of Boolean algebras which carry finitely-additive measures with different properties. These classifications will also be used to obtain a structure theory for such Boolean algebras. For a survey about these problems see Mirna Džamonja's paper "Measure recognition problem", published in the Philisophical Transactions of the Royal Society, 2006.

For papers regarding these topics, see my website:
http://www.logic.univie.ac.at/~thompson/pubs.html

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I am a second year Ph.D. student in University of Amsterdam and my supervisor is prof. Jouko Väänänen. The topic of my research is second order logic. The basis of my work is the following result of Ajtai[1] : It is independent of ZFC whether all second order equivalent countable models are isomorphic.

My best result so far is generalization of Ajtai's result to arbitrary successor cardinals using second order strengthenings of infinitary languages $L_{\kappa^+,\omega}$.

Using the idea of Ajtai's proof can be proved that if there is second (or respectively third, fourth ...) order definable well-order of the reals, then all second (or respectively third, fourth ...) order equivalent countable models are isomorphic. That is why I am interested in second order definable well-orders of the reals. Recently I have studied which large cardinals are consistent with second order definable well-orders of the reals.

When Ajtai proves that in L all second order equivalent countable models are isomorphic, he uses heavily the second order definable well-order of the reals in L. It would be interesting to know if there is a model of ZFC with

(1) There is no second order definable well-order of the reals.
(2) All second order equivalent countable models are isomorphic.

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My area of research is the Combinatorics of Singular Cardinals and more specifically Cardinal Arithmetic. In this area Shelah's PCF theory has become fundamental and the basic technical tool in this domain is the concept of the characteristic function of an internally approachable submodel.

A submodel always means a submodel of some large enough model $H(\Theta)$, where we usually add a predicate for a well-ordering thus automatically having access to Skolem functions and to Skolem Hulls of subsets of the model $H(\Theta)$. An uncountable submodel N is internally aproachable if it is the union of a continuous elementary chain of submodels and the initial segments of the chain belong to N. An important case is when N contains all of its subsets of size less than $|N|$. The characteristic function of a submodel N is the function that to every cardinal κ belonging to N attributes the value $\sup(N \cap \kappa)$. Another basic concept is the one of the set of μ's such that $\sup(N \cap \mu)$ belongs to the Skolem Hull of N united to the singleton $\sup(N \cap \kappa)$, this is the basic neighborhood of κ.

In Cardinal Arithmetic, Shelah's PCF conjecture has also become fundamental. There are two ways to attack this conjecture through the construction of internally approachable submodels whose characteristic functions satisfy certain properties of the sort: some cardinals being or not in the basic neighborhoods of other cardinals. The possible constructions of these submodels seem to be different from the known constructions of countable submodels that use games of length omega and trees of height omega, i.e, Namba combinatorics. To construct the mentioned internally apprachable submodels one is also allowed to use generic extensions by sufficiently distributive posets.

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I am interested in the interplay between set theory and algebra and its applications to the theory of modules over commutative rings. In particular my research focuses on structure theorems and realization theorems for Abelian Groups - a category that is usually a good test-class for possible results on larger modules classes.

(Abelian) Groups are defined by a set of elements and their defining relations. For instance, the well-known divisible Prüfer group $\mathbb{Z}(p^\infty)$ for some prime p is generated by elements $\{x_0, x_1, x_2, \cdots\}$ such that $px_0 = 0, px_1 = x_0, px_2 = x_1$. More generally, Crawley and Hales call a group simply presented if its defining relations are of the form $nx = y$ or $nx = 0$ where n is a positive integer. Obviously, there are much more complicated groups increasing the complexity of the relations. A group homomorphism is a map that respects those relations and very often homomorphisms and automorphisms of groups give you much information about the group itself. Thus the idea is to use combinatorial principles like the Black Boxes, properties of cardinals and a set-theoretic machinery to construct abelian groups (or modules over more general rings) having many, only a few or prescribed homomorphisms.

As an example transitivity properties of modules are subject of my studies. Transitivity, weak transitivity and full transitivity provide examples of modules with a rich structure and the property that every two elements can be mapped onto each other under certain natural and necessary assumptions. The notion of transitivity and full transitivity goes back to I. Kaplansky while weak transitivity is a new concept and is of a categorical nature. Similarly I am interested in the structure of the group of extensions Ext(G,H) for R-modules G and H. Since the solution of the famous Whitehead problem by Saharon Shelah it is well-known that this structure, in particular the vanishing of Ext(G,Z), depends on the underlying set theory. In some models a characterization of Ext(G,Z) is known - in others its understanding is beyond reach. Again, infinite combinatorics and set theory can be used to prove structure theorems like singular compactness theorems, the existence of universal modules etc. This has also applications in tilting and cotilting theory.

Since most of the constructions and techniques I am using involve (infinite) combinatorics, geometric objects and set-theoretic as well as model-theoretic arguments I am also very much interested in these areas, e.g. cardinal arithmetic and axiomatic set theory.

For my publications and further details on my research interests please see my webpage or contact me directly by e-mail: lutz@math.uni-bonn.de

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My research concentrates on descriptive set theory, including the structure of the real line and properties of Borel functions.

In a joint paper with Janusz Pawlikowski (*Two stars*, to appear) we investigate an operation * on the subsets of $\mathcal{P}(\mathbb{R})$ which is connected with Borel's strong measure zero sets as well as strongly meager. By definition, $\mathcal{A}^* = \{B \in \mathcal{P}(\mathbb{R})\colon \forall A \in \mathcal{A}\ A + B \neq \mathbb{R}\}$ for any $\mathcal{A} \subseteq \mathcal{P}(\mathbb{R})$. The results concern the behaviour of the family of countable sets when * is applied to it. We give a short proof of a theorem of Solecki stating that the family of countable subsets is a fixed point of ** (superposition of * twice). We also construct a translation invariant σ-ideal I such that I^* is equal to the family of countable subsets of $\mathbb{R}$.

Recently, I was interested in the structure of Borel functions which are not σ-continuous (i.e. there is no countable family of subsets (arbitrary) of it's domain such that the function is continuous on each of these sets). It was an old open problem solved by Keldiš, Adyan and Novikov if such functions exist. However, a particularly simple example was given by Pawlikowski: $P : (\omega+1)^\omega \to (\omega)^\omega$ is defined as $P(f)(n) = f(n) + 1 \pmod{\omega + 1}$. It turnes out (due to a theorem of Solecki) that this is actually the simples such function (it factorises every Borel not σ-continuous function). I use the determinacy of Borel games to get some additional results in this field.

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Singular cardinal combinatorics in the context of large cardinals
The driving interest of my researches has been the study of singular cardinal combinatorics in models of strong forcing axioms like MM or PFA. There are several problems in this area of which we get a clear picture assuming strong forcing axioms. In particular the techniques that led me to a proof of SCH from PFA are useful to study many other related issues:

- **Cardinal arithmetic.** The singular cardinal hypothesis SCH holds in models of MM or PFA and there are currently a number of proofs of SCH from MM or from PFA (for a proof of SCH from PFA see [3]).
- **The approachability ideal.** On one hand MM implies that there is a stationary subset of $S^{\aleph_{\omega+1}}_{\aleph_1}$ in the approachability ideal on $\aleph_{\omega+1}$. On the other hand, under MM, $\aleph_1$ is the unique cofinality for which the approachability ideal does not contain a relative club [2].
- **Saturation properties of models of strong forcing axioms.** We have the following results [3]:
  - If V is model of MM and W is an inner model with the same cardinals then $cf(\kappa)^W > \aleph_1$ if and only if $cf(\kappa) > \aleph_1$ for all cardinals κ.
  - if V models MM and is a set forcing extension of W and V and W have the same cardinals, then $[Ord]^{\aleph_1} \subseteq W$.

The above results suggest that the following should hold:

Conjecture 1. *Assume* MM *and let* W *be an inner model with the same cardinals. Then:*

(1) $[Ord]^{\aleph_1} \subseteq W$,
(2) κ *is regular iff* κ *is regular in* W.

A positive answer to these questions would suggest that a model of MM is essentially characterized by its cardinal structure, since any submodel which computes correctly the cardinals resembles closely to the universe.

A suitable version of the above results and conjectures can be stated also for supercompact cardinals.

I would like to continue to investigate the ground for the above conjectures and also to attack the problem of the eventual consistency of $\aleph_\omega$ being a Jónsson cardinal. My first step would be to try to prove that $\aleph_\omega$ is not Jónsson in a model of MM. König [1] has already shown that MM is consistent with $\aleph_\omega$ not being Jónsson.

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I am working under the supervision of professors Joan Bagaria and David Aspero (both from the University of Barcelona and ICREA). Our research deals with some topics in combinatorial set theory and specially with Jonsson cardinals.

We say that a cardinal κ is Jonsson if for every algebra of cardinality κ and with a countable number of operations, there exists a proper subalgebra of cardinality κ. The major problem in this area is to know whether it is consistent to assume that $\aleph_\omega$ is Jonsson. This question is interesting by itself but it also has deep connections with some model theoretic transfer properties. In fact, it is not difficult to see that the statement "$\aleph_\omega$ is Jonsson" is equivalent to an infinite version of Chang's conjecture.

In his doctoral thesis M. Foreman gave a first step in this direction by showing that the consistency of $\langle\aleph_{n+3},\aleph_{n+2},\aleph_{n+1}\rangle \twoheadrightarrow \langle\aleph_{n+2},\aleph_{n+1},\aleph_n\rangle$ follows from the existence of a 2 huge cardinal. Our project intends to improve this result and it is actually centered on a possible scenario which would give the consistency of $\langle\aleph_{n+k+1},\aleph_{n+k},\ldots,\aleph_1\rangle \twoheadrightarrow \langle\aleph_{n+k},\aleph_{n+k-1},\ldots,\aleph_0\rangle$, for all finite numbers n and k.

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I am interested in large cardinals and combinatorial set theory. Past work has included the following topics:

(1) Several papers on Boolean algebras. These focus on complete embeddings of the Cohen algebra into possible candidates for a counterexample to von Neumann's problem in ZFC; and games related to generalized distributive laws), co-stationarity of the ground model (investigating when it is possible for the $\mathcal{P}_\kappa\lambda$ of the ground model to be co-stationary in a forcing extension). (One of these papers is joint with James Cummings.)

(2) Co-stationarity of the ground model. These papers focus on the question of when is it possible, or necessary, that adding a new set of some kind (for instance, a new subset of $\aleph_1$) will make the $\mathcal{P}_\kappa\lambda$ of the extension model so much larger than the $\mathcal{P}_\kappa\lambda$ of the ground model that the collection of new sets in $\mathcal{P}_\kappa\lambda$ in the extension model is stationary. (Two out of three of these papers are joint with Sy-David Friedman.)

(3) The tree property. Last year we proved that the tree property at the double successor of a measurable is equiconsistent with a weakly compact hypermeasurable cardinal. I am interested in related questions about the tree property at the double successor of other large cardinals, or simlutaneously for class many cardinals, etc. (This paper is joint with Sy-David Friedman.)
(4) Some work in recursion theory with Steve Simpson on recursion theoretic versions of the set theoretic properties of measure algebras.

In addition to these topics, I am currently interested in any problems involving partial orderings, trees (particularly Aronszajn or Suslin trees), Boolean algebras, and set-theoretic topology in general.

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My research area is in Generic Absoluteness, more specifically on Projective Absoluteness and its relations with Descriptive Set Theory. My advisor is Joan Bagaria from the University of Barcelona and ICREA.

Projective Absoluteness studies the invariance of the truth values of the statements about real numbers between a model and its generic extensions.

Formally, for a model M of ZFC^*, $n \in \omega$, and a poset $\mathbb{P}$, we say that M is $\boldsymbol{\Sigma^1_n}$-$\mathbb{P}$-*absolute* if $M \prec_{\Sigma^1_n} M^{\mathbb{P}}$, i.e., the truth value of every Σ^1_n-formula is invariant by forcing with $\mathbb{P}$, that is, for any Σ^1_n-formula $\varphi(x)$, and for every real $a \in M$, $M \models \varphi(a)$ iff $M^{\mathbb{P}} \models \varphi(a)$.

Shoenfield [6] proved in ZFC that the truth value of every Π^1_2-statement (the negation of a Σ^1_2-statement) with real parameters is invariant under forcing, that is, $\boldsymbol{\Pi^1_2}$-$\mathbb{P}$-absoluteness holds for every poset $\mathbb{P}$. Since the statement which says "*there is a nonconstructible real*" can be formalized by a Σ^1_3 formula and it is false in L but can be forced to be true, Σ^1_3-absoluteness cannot be proved in ZFC. Indeed, the consistency strength of $\boldsymbol{\Sigma^1_3}$-absoluteness is a reflecting cardinal [2, 3].

Surprisingly, Projective Absoluteness is strongly connected with the topological regularity properties of projective sets of reals, those sets definable by some projective formula (Σ^1_n or Π^1_n) by means of real parameters. These are properties that assert that the sets resemble in some topological aspect very much to a Borel set and, in this sense, that they have a nice behaviour.

By results of Bagaria [1], Judah and Shelah [5], and Ikegami [4], we know that for some natural forcing notions $\mathbb{P}$, there is a regularity property $P_{\mathbb{P}}$ of sets of reals such that the following are equivalent:

(1) $\boldsymbol{\Sigma^1_3}$-$\mathbb{P}$-absoluteness,
(2) Every $\boldsymbol{\Delta^1_2}$ set of reals has the property $P_{\mathbb{P}}$,

where a set of reals is $\mathbf{\Delta^1_n}$ if it can be defined at the same time by a Σ^1_n-formula and by a Π^1_n-formula by means of real parameters.

On the other side, Feng, Magidor, and Woodin proved in [3] that the following are also equivalent:

(1) $\mathbf{\Sigma^1_3}$-$\mathbb{P}$-absoluteness for every set forcing $\mathbb{P}$,
(2) Every $\mathbf{\Delta^1_2}$ set of reals is universally Baire,

where the universal Baireness is a topological property which implies all the classical regularity properties. A set is *universally Baire* if for every infinite cardinal κ, for every continuous function $f\colon \kappa^\omega \to \omega$, $f^{-1}[A]$ has the Baire property in κ^ω. Universal Baireness allows one to describe a set with this property using trees whose projections are complementing each other in any generic extension.

My research goal now is to study if there is some equivalence of this kind for every forcing $\mathbb{P}$, i.e., isolate for every forcing $\mathbb{P}$ (or for some specific classes of forcings such as ccc, proper, or semiproper, for example) some topological property inspired in universally Baireness which could give this kind of equivalence.

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My mathematical interests focus on the Set Theory. In particular its descriptive part, method of forcing and their applications to the survey of the real line lie in the center of my studies.

My first area of interest are Definable Equvalence Relations. I am interested in Borel equivalence relations and its reducibility properties. Recently, analyzing E_1 ER I obtained a simpler proof of a theorem on illfounded Sacks iteration. I also work on Real Ordinal Definable relations in the Solovay's model. In particular I study a so-called ROD-diagram in which me and prof. Kanovei have recently found some nice new properties. In particular quite sophisticated methods, as a selection theorem by Solovay, are used to prove these properties. As a byproduct we learn how to live in the Solovay model which recently becomes a nice tool to study iterated forcing.

Second area of my research concentrates on a geometric language which can be used to describe a family of forcing models. The models are produced by forcing with posets containg positive Borel sets, where positive means outside of a well-descriptive-definable (e.g. Π^1_1 on Σ^1_1) sigma ideal. In particular using a descriptive and geometric analysis of forcing conditions in standard Cohen generic extensions me and J. Pawlikowski showed how to extend Baire Property by uncountably-many sets obtaining a category counterpart of Carlson research.

While working with classical forcing iterations I am also interested in an illfounded Sacks' iteration, studied by Kanovei. It turned out that methods of so-called iterated perfects, which are directly connected with the geometrical properties of the abovementioned forcing conditions, can be used here as well. Using them I obtained a simpler proof of one of the Kanovei theorem I have written above.

The last field of my interest I mention here is a version of the Covering Property Axiom (by Ciesielski and Pawlikowski) for Miller and Laver forcings. The original version of this axiom encaptures a majority of properties which hold in the iteration of the Sacks forcing of length ω_2. I hope to obtain a similar version of the axiom for the two iterations I mentioned above.

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Axiomatic set theory: determination of consistency strengths of infinitary combinatorial principles, using forcing and core models; characterizations of large cardinal axioms by embeddings of models of set theory.

Constructibility theory and hyper computations: new fine structure theories for constructible models of set theory, with applications; generalized machines with tapes of arbitrary ordinal lengths or registers working on ordinal numbers.

Descriptive set theory and infinitary games: representation of sets of reals by systems of models of set theory, an alternative proof of the Martin-Steel projective determinacy theorem.

Editor for descriptive set theory in the edition of the collected works of Felix Hausdorff.

General logic: a computer-checked formal proof of Gödel's completeness theorem; NAPROCHE - design of a proof checking systems with natural language interfaces, in collaboration with linguistics.

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My research area is descriptive set theory and especially applications of inner model theory and large cardinals to descriptive set theory. My doctoral thesis under the supervision of Ralf Schindler is concerned with projective equivalence relations and equivalence relations in $L(\mathbb{R})$ under determinacy.

An equivalence relation on Baire space is called thin if there is no perfect set of pairwise inequivalent reals. For example any $\utilde{\Sigma}^1_{2n}$ norm defines a $\utilde{\Pi}^1_{2n}$ equivalence relation and this is thin if $\utilde{\Delta}^1_{2n}$ determinacy holds. Thin projective equivalence relations are closely connected with the projective ordinals, for instance it follows from a classical result of Harrington and Shelah that the maximal number of equivalence classes of thin $\utilde{\Pi}^1_{2n}$ equivalence relations is $\utilde{\delta}^1_{2n-1}$ assuming $AD^{L(\mathbb{R})}$.

A natural goal is to characterize those inner models which have representatives for all equivalence classes of thin equivalence relations of a given complexity. In my thesis I study the property for an inner model M: M has representatives for all equivalence classes of all thin $\utilde{\Pi}^1_{2n}$ equivalence relations defined from a parameter in M. Greg Hjorth proved that an inner model M has this property for $n = 1$ if and only if M is Σ^1_3-correct in V and $\omega_1^M = \omega_1^V$, assuming all reals have sharps. I generalized this result to $n \geq 1$ with a different proof technique using iterable premice with Woodin cardinals. On higher levels one has to use the corresponding level of correctness and consider the tree from a canonical Π^1_{2n-1} scale instead of ω_1. Currently I am trying to extend this result to higher scaled pointclasses in $L(\mathbb{R})$.

I am also interested in the Wadge order for Polish spaces. For many Polish spaces the Wadge order (given by the continuous maps) is more complicated than for Baire space under AD. For spaces with a nontrivial connected subset I proved that there is an antichain of size continuum. Another result is that the Wadge order for the real line contains a copy of $(\mathcal{P}(\omega), \subseteq_{fin})$ and an antichain of size $\theta^{L(\mathbb{R})}$ in $L(\mathbb{R})$. There are many interesting open questions on this topic.

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I'm interested in applications of set theory to measure theory, topology and functional analysis. Below I list my current areas of interest.

Measure games. In [7] Fremlin introduced infinite games defined in the same way as Choquet (Banach–Mazur) games in topology, but in which players use elements of a σ–algebra of measurable sets instead of open sets. Among other results Fremlin showed that if we play Borel subsets of a Polish space, then the second player has a winning strategy. In [1] Grzegorz Plebanek and I presented an easier and more intuitive proof of this fact. One of the interesting open problems in this field is: is there a measure space such that the second player has a winning strategy but has no winning tactic? Debs in [4] showed an example of topological space with this property.

Counterexamples via Stone isomorphism. There is a method of constructing peculiar topological (or Banach) spaces using Boolean algebras. The idea is to encode some combinatorial properties in a Boolean algebra (often, a subalgebra of $P(\omega)$) to obtain an interesting topological space as a Stone space of this algebra. Longstanding Efimov problem is a good example of a problem, which could be solved by this method. Efimov asked if there is an infinite compact space without nontrivial convergent sequences and without a copy of $\beta\omega$ (a *Efimov space*). There are partial (negative) answers to this problem, e.g. under CH or a certain assumption on the splitting number $\mathfrak{s}$. In [2] I give some constructions (in ZFC and under MA) of spaces satisfying certain Efimov–like properties. In [3] Grzegorz Plebanek and I showed that the existence of Banach space which has a Mazur Property and does not have a Gelfand-Phillips property (properties connected to the theory of Pettis integrals, which have something to do with convergence of sequences of points and of measures) is equivalent to certain cardinal invariants inequality. One of the tools used in constructions of peculiar compact spaces via Stone isomorphisms is *minimally generated Boolean algebra* (see [8]). They were used in [5], [9] for constructions of Efimov spaces. I investigated (mainly measure theoretic) properties of minimally generated Boolean algebras in [2].

Separable measures. There is a well–known question (MRP(separable) in terms of [6]) about a characterization (combinatorial, topological) of the class of Boolean algebras admitting only separable measures. In [2] I showed that every Boolean algebra either admits a uniformly regular measure or it carries a measure which is non–separable and that the class of minimally generated Boolean algebras is a (quite rich) class of Boolean algebras which carry only separable measures.

The combinatorial structure of $P(\omega)$. Seeking interesting examples of topological spaces via Stone isomorphism we usually need to solve certain combinatorial problems concerning the structure of $P(\omega)$, in particular we have to deal with inequalities on cardinal invariants (such as $\mathfrak{p}$, $\mathfrak{b}$, $\mathfrak{h}$). In my work the cardinal invariants connected to the ideal of asymptotic density zero sets are particularly important (see [3]).

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My current work concerns the restrictions one has to put on the *Easton function*, or *continuum function*, on regular cardinals in the context of large cardinals with reflection properties (this would typically be measurable cardinals). More specifically, if F is an Easton function, i.e. for all regular cardinals α, β we have $\alpha < \beta \to F(\alpha) \le F(\beta)$ and $\mathrm{cf} F(\alpha) > \alpha$, we ask which cardinals κ remain measurable in a cofinality-preserving generic extension realizing F, i.e. $2^\alpha = F(\alpha)$ for α regular. The potential preservation of measurability of κ while the its power set has a prescribed value $F(\kappa)$ allows for a subsequent singularization via a single Prikry sequence, obtaining a failure of SCH in the context of the given Easton function F.

By results of Gitik, if κ is measurable and $2^\kappa = \lambda$, we need at least the strength of $o(\kappa) = \lambda$, which is slightly weaker than κ being λ-hypermeasurable (this means that $H(\lambda)$ of V is included in a target model of some elementary $j : V \to M$ with critical point κ). However, it seems that to obtain $2^\kappa = \lambda$ while keeping κ measurable *and simultaneously* realizing an arbitrary F on all regular cardinals, we need the full strength of λ-hypermeasurability.

In particular, we have shown[4] that if F is an Easton function, then there is a cofinality-preserving generic extension V^* of V which preserves measurability of every κ satisfying the following single non-trivial condition:

- κ is $F(\kappa)$-hypermeasurable in V and this is witnessed by an embedding $j : V \to M$ such that $j(F)(\kappa) \geq F(\kappa)$.

Building on a work by Menas, we have also shown that if F is simply defined, then all strong cardinals are preserved in the generic extension V^*.

Future work might inquire whether one really needs the full strength of $F(\kappa)$-hypermeasurability in the above result, or what other large cardinals may be preserved. A more difficult question would be to what extent such results can be extended to Easton functions defined on singular cardinals as well.

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Left distributive algebras (LD's) are sets with one binary operation satisfying the law $a(bc) = (ab)(ac)$. The most common LD's are group conjugation and the weighted mean ($x * y = px + (1-p)y$), each of which occur in the study of knots and braids. Beginning in the late 1980's, an interesting connection was discovered between free left distributive algebras and large cardinal axioms. While group conjugation and the weighted mean are left distributive, they are idempotent, hence not free. We denote the free left distributive algebra on one generator by $\mathcal{A}$ and the free LD on κ generators by $\mathcal{A}_\kappa$. $\mathcal{A}$ manifests fairly naturally in two contexts: as an algebra which exists under the assumption of the existence of a nontrivial rank embedding (a very strong large cardinal hypothesis), and as a particular operation on a subset of the braid group B_∞.

Define $p <_L q$ for $p, q \in \mathcal{A}$ if and only if q can be written as $(((pq_1)q_2)\cdots q_n)$, for some $q_1, q_2, \ldots q_n$. Laver [2] and Dehornoy [1] proved independently and by different methods that $<_L$ linearly orders $\mathcal{A}$. Laver's method demonstrated the linearity of $<_L$ by establishing the existence of a division form for the elements of $\mathcal{A}$, and this division form has consequences for the structure of $\mathcal{A}$. The division algorithm itself takes place in a larger algebra $\mathcal{P}$ that is formed by freely adding a composition operation, $\circ$, to $\mathcal{A}$.

The original proof of the division form theorem for $\mathcal{P}$ relies on the existence of another normal form theorem of Laver. I have given a new proof that establishes the result directly in the hopes that it will be more useful in generalizing the division form theorem to the many-generator case. There are significant complications that arise when considering terms in $\mathcal{A}_\kappa$ as

[4] Sy D. Friedman and Radek Honzik. Easton's theorem and large cardinals. Submitted to APAL.

opposed to the one generator case; as the κ distinct generators of $\mathcal{A}_\kappa$ are unordered by $<_L$, there is necessarily a new obstacle to comparing words. I hope to soon complete the generalization to achieve that every word in $\mathcal{P}_\kappa$ has a division form equivalent.

A related problem that I would be interested in pursuing at the workshop is that of demonstrating the existence of a copy of $\mathcal{A}_2$ from the assumption of the existence of a nontrivial embedding $j : V_\lambda \to V_\lambda$. I have considered a pair of embeddings generated from j such that I believe neither can be generated from applications of the left distributive law to the other, but I've not been able to prove it.

Most of the other problems I am working on (for example, various well-foundedness questions) are related to LD's, and are essentially algebraic in nature. I don't know that anyone else would be interested in them, though if they are I will happily discuss them!

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My research centers around applications of pure set theory and descriptive set theory in real analysis. In my doctoral thesis I study descriptive complexity of sets which appear naturally in real analysis. I show that set of strictly singular autohomeomorphisms of the unit interval is Π^1_1–complete, in particular non–Borel. I investigate descriptive complexity of compact subsets of real line with prescribed Lebesgue density and porosity, and set of all functions differentiable on co-countable sets in $C[0, 1]$. I am also interested in studying of properties of small sets in Polish spaces.

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My primary research interest is **descriptive set theory**. In my PhD thesis I developed a new concept in the theory of **Hurewicz tests**. Roughly speaking, a Hurewicz test allows to witness the class of sets in the Borel hierarchy. My main revelation concerns the Hurewicz tests for the Borel class Σ^0_ξ $(0 < \xi < \omega_1)$, as follows. For every fixed ordinal $\xi < \omega_1$ it is possible to endow the Polish space 2^ω with a finer Polish topology in such a way that every Σ^0_ξ is either "almost empty" or of second category in this finer topology. This makes possible the application of the Baire Category Theorem for Σ^0_ξ sets. Accordingly, this concept turned out to have **numerous Baire Category Theorem-like applications**. In particular,

- it implies that if the union of less than $\mathrm{cov}(\mathcal{M})$ many Σ^0_ξ sets is Borel then it is Σ^0_ξ (see [6]); this reproves a theorem of J. Stern.
- it allows the construction of Hurewicz test sets for generalized separation of analytic sets (see [3]).
- under the assumption of the continuum hypothesis, it refutes a question of A. Miller concerning analytic ideals (see [7]); the question had previously been refuted in an unpublished work of A. Kechris and M. Zelený using $V = L$;
- using these test pairs it is possible to show that there is no monotone presentation for Borel sets (see [5]); this result is a natural counterpart of some results of M. Elekes and A. Máthé, and of A. Andretta, G. Hjorth and I. Neemann.

Currently I am interested in the study of **σ-ideals of compact** sets and of **Borel equivalence relations**. I managed to answer in the negative a question of A. Kechris by constructing a G_δ σ-ideal of compact subsets of 2^ω which contains all the singletons but does not contain all the compact subsets of any dense G_δ set in 2^ω (see [4]). This result indicates that G_δ σ-ideals can exhibit wilder behavior than expected. Concerning Borel equivalence relations, currently I study relations E satisfying $l^1 \leq_B E \leq_B l^\infty$. In our joint work with M. Vizer we obtained that, contrary to present-day belief, there are many other Borel equivalence relations E satisfying $l^1 \leq_B E \leq_B l^\infty$ than just the l^p ones or the direct sums of these (see [8]).

As a member of the set theory and general topology workgroup of the Rényi Institute, led by I. Juhász, I am active in **set theoretic topology**, as well. During a research stay in the workgroup of D. Preiss at University College London I worked on l-equivalence of topological spaces (see [2]). In my recent joint work with M. Elekes and L. Soukup we examined the problem whether for some cardinals κ and λ, in a given topological space, a κ-fold cover by sets with various geometric and topological properties can be split into λ many disjoint subcovers (see [1]).

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Fields: Forcing, Descriptive Set Theory, Set Theory of the Reals.

In *Set Theory of the Reals*, one studies the connection between certain properties of sets of reals like *regularity properties* (e.g. Baire property, Lebesgue measurability) and the *complexity* of sets. For example, every analytic set has the Baire property and is Lebesgue measurable. But when we replace "analytic" with a higher complexity level like $\mathbf{\Sigma}_2$ or $\mathbf{\Delta}^1_2$, the issue becomes meta-mathematical and the theory of forcing comes in. A typical such example is Solovay's characterization theorem [Sol69] which states:

"Every $\mathbf{\Sigma}_2$ set is Lebesgue measurable $\iff \forall x$ (the set of random reals over $\mathbf{L}[x]$ is null)"

We can generalize this result as follows: if $\mathbb{P}$ denotes an arbitrary forcing partial order, we associate to it an *algebra of measurability* $\mathcal{A}(\mathbb{P})$ as well as a *null-ideal* $\mathcal{I}(\mathbb{P})$. A Solovay-style theorem then reads as follows:

"Every $\mathbf{\Sigma}_2$ set is in $\mathcal{A}(\mathbb{P}) \iff \forall x$ (the set of $\mathbb{P}$-generic reals over $\mathbf{L}[x]$ is in $\mathcal{I}(\mathbb{P})$)"

Other variants of such results, also called *Judah-Shelah-style theorems*, are

"Every $\mathbf{\Delta}^1_2$ set is in $\mathcal{A}(\mathbb{P}) \iff$ for every x, there is a $\mathbb{P}$-generic real over $\mathbf{L}[x]$"

Such theorems have been proved for different $\mathbb{P}$, among others in [JuShe89, BrLö99, BrHaLö05].

The aim of my research is to generalize these theorems in such a way as to cover a wide variety of forcing notions. One direct apporach is finding conditions on $\mathbb{P}$, so that if those are satisfied, then a Solovay-style or a Judah-Shelah-style theorem can be proved for $\mathbb{P}$. Another approach is via *cardinal invariants*: for example, if $\mathsf{add}(\mathbb{P}) \leq \mathsf{add}(\mathbb{Q})$ then this gives a good indication that a Solovay-style theorem for $\mathbb{P}$ implies one for $\mathbb{Q}$. The goal of my project is to make these "indications" precise and find out more about the underlying theory.

A starting point for my research is Zapletal's analysis of a similar problem in [Z04], as well as some ideas introduced in [BrHaLö05]. Also, this project may potentially be carried out in partial collaboration with my fellow PhD student Daisuke Ikegami, who has already obtained new results in this direction [Ik06].

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Conference picture

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