Research Statement

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I0, i.e the existence of an elementary embedding j from $L(V_{\lambda+1})$ to itself, for some $\lambda > \mathrm{crt}(j)$, has been considered for some time the strongest large cardinal hypothesis. The appeal of I0 consists in the particular features shared with $\mathsf{AD}^{L(\mathbb{R})}$, like measurability or a generalization of the Coding Lemma. Woodin in his future latest book introduced new large cardinal hypotheses, stronger than I0, and proved similar results of affinity. The idea is that between $L(V_{\lambda+1})$ and $L(V_{\lambda+2})$ there are a lot of intermediate steps, namely $L(N)$ with $V_{\lambda+1} \subset N \subset V_{\lambda+2}$, with the particular case $L(X, V_{\lambda+1})$ with $X \subset V_{\lambda+1}$. Trying to find an equivalent of $\mathsf{AD}_{\mathbb{R}}$, Woodin defined a sequence of $E^0_\alpha(V_{\lambda+1})$, in a similar way of the minimum model of $\mathsf{AD}_{\mathbb{R}}$, such that $V_{\lambda+1} \subseteq E^0_\alpha(V_{\lambda+1}) \subseteq V_{\lambda+2}$, $L(E^0_\alpha(V_{\lambda+1})) \cap V_{\lambda+2} = E^0_\alpha(V_{\lambda+1})$, there always exists an elementary embedding from $L(E^0_\alpha(V_{\lambda+1}))$ to itself and when the sequence ends in the "right way" (if that is consistent), then $L(\bigcup E^0_\alpha(V_{\lambda+1}))$ has characteristics similar to a model that satisfies $\mathsf{AD}_{\mathbb{R}}$.

My attention is focused on the properties of the elementary embeddings from $L(E^0_\alpha(V_{\lambda+1}))$ to itself, expecially when $L(E^0_\alpha(V_{\lambda+1})) \vDash V = \mathsf{HOD}_{V_{\lambda+1}}$. One of the key properties in this setting is *properness*, that is a fragment of the Axiom of Replacement for the elementary embedding. Properness implies iterability and many other similarities wirh $\mathsf{AD}^{L(\mathbb{R})}$, but it seems almost trivial, since the most immediate examples of elementary embeddings satisfy it. In my PhD thesis, in collaboration with Woodin, I've proved that in fact if the sequence is long enough (for example when it ends in the right way) then there exists an α such that every elementary embedding from $L(E^0_\alpha(V_{\lambda+1}))$ into itself is not proper. It is also possible to find an $L(E^0_\beta(V_{\lambda+1}))$ that contains both proper and non-proper elementary embeddings, so properness is not a property directly implied by the model.

In the future I want to approach the hypothesis that the $E^0_\alpha(V_{\lambda+1})$-sequence does *not* end in the right way. This would imply that the $E^0_\alpha(V_{\lambda+1})$-sequence is a standard representative for all the $L(X, V_{\lambda+1})$ that contain elementary embedding, and a preliminary study seems to indicate that this will give results on the properness of elementary embeddings from $L(X, V_{\lambda+1})$ to itself, that is still an open problem. Since this is still a virgin territory (even because it's based on unpublished results), the possible outcomings are many.