Recently I've focused on strong forcing axioms like PFA and MM, particularly their effect on generic embeddings of V with critical point ω_2. My interest in this subject was motivated by two condensation-like consequences of forcing axioms. Here is a very imprecise formulation of these results (if $M \prec H_\theta$ then $\sigma_M : H_M \to M$ denotes the inverse of the Mostowski collapse of M):

(1) (Foreman [3]) Assume MM. Then there are stationarily many $M \in [H_\theta]^{\omega_1}$ for which H_M is correct about a large portion of $NS \upharpoonright \mathrm{cof}(\omega)$.

(2) (Viale-Weiss [5]) Assume PFA. Then there are stationarily many $M \in [H_\theta]^{\omega_1}$ such that, if $F : [H_\lambda]^{\omega_1} \to V$ is a *slender function* and $F \in M$, then $\sigma_M^{-1}[F(M \cap H_\lambda)]$ is an element of H_M.

In [1] I strengthened Foreman's result and introduced the *Diagonal Reflection Principle (DRP)*, which is a highly simultaneous form of stationary set reflection. Similar results were independently obtained by Viale [4]. DRP has several convenient characterizations; one characterization states that the forcing with positive sets for $NS \upharpoonright \wp_{\omega_2}(\theta)$ has a property resembling but weaker than properness (namely, that stationary subsets of $[\theta]^\omega$ remain stationary in $ult(V, G)$ though not necessarily in $V[G]$). This prompted the natural question of whether PFA can co-exist with ideals on ω_2 whose associated posets are proper; in [2] I showed that this is possible, starting from a superhuge cardinal.

There are natural strengthenings of PFA (which hold in the model from [2]) which imply there are generic embeddings of V where a large portion of the embedding is an element of V. This uses DRP and ideas from [3]. I am currently exploring this further with Matteo Viale and Christoph Weiss, and am also looking into topological applications of DRP.

References

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