Raphaël Carroy, Université de Lausanne and Paris 7

I am interested in descriptive set theory, and more specifically the study of measurable functions. There is two ways to define a hierarchy of functions. On the first hand, the Baire hierarchy of functions is defined inductively from the continuous functions using the taking of pointwise limit of a sequence of functions. On the other hand, the Borel hierarchy of functions is based on the complexity of inverse images of open sets in the Borel hierarchy of sets. Lebesgue has showed an exact correspondence between these two hierarchies.

A space is *polish* whenever it is separable and completely metrizable. In other words, when it admits both a metric such that every cauchy sequence converges and a countable dense subset. Given A and B two polish spaces, a function $f : A \to B$ is *of Baire class one* if f is the pointwise limit of a sequence of continuous functions, or equivalently if the inverse image of an open, or Σ^0_1 subset of B is a countable union of closed sets, or Σ^0_2, in A.

For a specific subset of the first Baire class there is a result binding complexity in terms of inverse images and partitions in continuous functions. Indeed the Jayne-Rogers theorem states that for A and B polish and $f : A \to B$ a function, the inverse image of a Σ^0_2 is Σ^0_2 if and only if there is a countable partition $(A_i)_{i\in\mathbb{N}}$ of A in closed sets such that for all $i \in \mathbb{N}$ the restriction of f to A_i is continuous.

Following this result Andretta proved that for A and B totally disconnected polish spaces, Baire class one functions can be represented as strategies in infinite games. Duparc found a game characterisation for the first Baire class of functions, and Semmes extended those results to the second Baire class.

Is it possible to extend those results to Borel functions of finite rank? Or to any Borel function? Is it possible to refine even more this hierarchy of functions, by looking for example at the Wadge degree of inverse images of open sets? Or by defining a notion of reduction for functions? This is some of the questions I am working on.

References

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