Marcin Sabok (UIUC, IMPAN, UWr)
www.math.uni.wroc.pl/~sabok

Research Statement

I am especially interested in the theory of *idealized forcing*. This is a topological and descriptive analysis of forcing notions of the form $P_I = \mathrm{Bor}(X)/I$, where X is a Polish space and I is a σ-ideal on it. There are two main directions in which the theory has made major progress. The first one develops forcing techniques that are used to force various behaviours of cardinal invariants and other phenomena on the real line. And the second one which applies forcing and absoluteness techniques to obtain new results in descriptive set theory.

One of the recent developments in the second direction (my joint work with Jindra Zapletal [2]) gives a new, purely descriptive set-theoretical dichotomy, which essentially uses idealized forcing. The result says that among σ-ideals generated by closed sets there are only two, drastically different cases which can occur. We say that a σ-ideal I has the *1-1 or constant* property if every Borel function defined on a Borel I-positive set can be restricted to a Borel I-positive subset, on which it is either 1-1 or constant. This is equivalent to saying that the forcing adds a minimal real degree. Of course, if P_I adds a Cohen real, then any name for it can be translated to a Borel function which cannot be restricted to be either 1-1 or constant. Now, the result says that a Cohen real is the only obstacle: for any σ-ideal I generated by closed sets, either P_I adds a Cohen real, or else I has the 1-1 or constant property. In many cases it is relatively easy to exclude the Cohen real (ω^ω-bounding, Laver property, etc.) and the 1-1 or constant property is a strong and useful statement. It can be also treated as a canonization result for smooth equivalence relations, i.e. saying that such equivalences trivialize after restriction to a Borel I-positive set. Now, there is a natural question for which σ-ideals and which equivalences the canonization can be obtained. This is work in progress, joint with Vladimir Kanovei and Jindra Zapletal.

Another interesting development in descriptive set theory, motivated by idealized forcing, is a recent result about the complexity of Ramsey-null sets. More precisely: the complexity of codes for analytic Ramsey-null sets. In many arguments in idealized forcing (esspecially about the iteration) it is important that the σ-ideal is absolutely definable. In most cases the σ-ideal is definable by a $\mathbf{\Pi}^1_1$ (or at least $\mathbf{\Delta}^1_2$) formula, which is absolute for countable models. A question that was first raised in idealized forcing (it was also asked independently by Daisuke Ikegami), is whether the σ-ideal of the Mathias forcing is definable in such way. In a recent result [1], I showed that it is not, and the reason is interesting from the descriptive set-theoretical point of view. It turns out, that the set of codes for Ramsey-positive analytic sets is $\mathbf{\Sigma}^1_2$-complete and this is a surprising analogon of the same phenomenon on the lower level of the projective hierarchy: the set of codes for uncountable (i.e. Sacks-positive) analytic sets is $\mathbf{\Pi}^1_1$-complete, which is an old and classical theorem of Hurewicz.

References

[1] Sabok M., *Complexity of Ramsey-positive sets*, submitted,

[2] Sabok M., Zapletal J.,*Forcing properties of ideals of closed sets*, Journal of Symbolic Logic, to appear.