Research Statement

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My current research focuses on questions concerning ultrafilters. Recently, new connections have been discovered between strong P-points and Mathias forcing and several interesting questions remain open, e.g.

Question Is is the relativized ultrafilter-Mathias forcing almost ${}^{\omega}\omega$-bounding provided the ultrafilter is a strong P-point?

Question Is the iteration of the natural forcing to add a strong P-point followed by killing this ultrafilter via Mathias forcing Π^1_1 on Σ^1_1?

Question Is being a Canjar filter equivalent to being a strong P-filter for meager-filters?
Of course, there are also other very intersting questions, which are probably much harder:

Question [1] Is it provable in ZFC that there is a non-meager P-filter?

One can also look at ultrafilters from a topological perspective and find interesting questions. For example there are points in ω^* which are limit points of a countable set without isolated points, however the neibourghood traces on any countable subset generate an ultrafilter. Put in a different way, these points cannot be $\mathcal{F}$-limits of a countable sequence for any filter which is not an ultrafilter. It is a standard fact that no point in ω^* can be a $\mathcal{FR}$-limit of a sequence. Recently I learned from T. Banach, that there are points, which can be $\mathcal{F}$-limits of some sequence with $\mathcal{F}$ a meager filter. This cannot happen if $\mathcal{F}$ is an F_σ-filter or an analytic P-filter.

Question Is there an analytic filter $\mathcal{F}$ such that the $\mathcal{F}$-limit of some sequence in ω^* exists?

References

[1] Kanamori, A. *Some combinatorics involving ultrafilters*, Fund. Math., vol 100, 1978, pp.145–155.