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Research statement

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My interests in set theory are both mathematical and philosophical. I'm attending my second year of PhD and since now I am focusing on the study of some consequences of the Forcing Axioms. In particular, I studied the problems related to Shelah's Conjecture (: every Aronszajn line contains a Countryman suborder) and the equivalent Five Element Basis Conjecture (: the orders $X, \omega_1, \omega_1^*, C$ and C^* form a five element basis for the uncountable linear orders any time X is a set of reals of cardinality $\aleph_1$ and C is a Countryman suborder). Moore showed that the Conjectures follow from PFA, but soon after has been discovered by König, Larson, Moore and Veličković that the consistency strength of the hypothesis can be reduced to that of a Mahlo cardinal, instead of that of PFA, whose upper bound is a supercompact cardinal and whose lower bound is a class of Woodin cardinals. Last year, Boban Veličković and I ([Veličković, Venturi 2010]), managed to give a more direct proof of the five element basis theorem, but still with the same hypotheses as in [König, Larson, Moore, Veličković 2008]

There are many problems related to this subject, that would be worth studying. First of all, a question that arise naturally is: do we really need some large cardinal strength for Shelah's Conjecture? Moreover it is interesting to see which are the influences of this Conjecture on the cardinality of the Continuum, because in the models of PFA, $2^{\aleph_0} = \aleph_2$, but if we do not need the consistency strength of PFA, can we find a model where Shelah's Conjectures holds, but the cardinality of the Continuum is difffers from $\aleph_2$?

Another subject I am interested in is the study of forcing of size $\aleph_1$. It has been proved, by Aspero and Mota, that PFA restricted to posets of size $\aleph_1$ is consistent with the continuum large. It would be interesting to see if it is possible to give a classification of this class of posets in the same way, under PFA, it is possible to classify the Aronszajn lines under the relation of be-embeddability.

References

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[Todorčević 2001] Stevo Todorčević : *Lipszhitz maps on trees*, Report 2000/01 number 13, Istitut Mittag-Leffler.

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