Research Statement

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My interest in set theory is **descriptive set theory**, especially determinacy, forcing absoluteness, and their connections with large cardinals and inner model theory. Currently, I am mainly working on **Blackwell determinacy** and its connection with **Gale-Stewart games**. Blackwell games are infinite games with imperfect information generalizing the game "Rock-Paper-Scissors" and Blackwell determinacy is an extension of von Neumann's minimax theorem for Blackwell games while Gale-Stewart games are infinite games with perfect information generalizing the game "Chess" and the determinacy of Gale-Stewart games has been deeply investigated in set theory.

In 1998, Martin proved that the Axiom of Determinacy (AD) implies the Axiom of Blackwell determinacy (Bl-AD) and conjectured the converse, which is still open to be true. In 2003, Martin, Neeman, and Vervoort proved that AD and Bl-AD are equiconsistent. Recently, with de Kloet and Löwe, I introduced the Axiom of Real Blackwell determinacy (Bl-$\mathrm{AD}_{\mathbb{R}}$) and proved that Bl-$\mathrm{AD}_{\mathbb{R}}$ implies the consistency of AD, so by Gödel's Incompleteness Theorem, the consistency of Bl-$\mathrm{AD}_{\mathbb{R}}$ is strictly stronger than that of AD.

Currently I am working with Woodin on the connection between Bl-$\mathrm{AD}_{\mathbb{R}}$ and the Axiom of Real Determinacy ($\mathrm{AD}_{\mathbb{R}}$). We are about to prove that they are equivalent assuming the Axiom of Dependent Choice (DC) and are working on whether they are equiconsistent. (Note that $\mathrm{AD}_{\mathbb{R}}$+DC implies the consistency of $\mathrm{AD}_{\mathbb{R}}$ by the result of Solovay. So the equivalence of $\mathrm{AD}_{\mathbb{R}}$ and Bl-$\mathrm{AD}_{\mathbb{R}}$ under DC does not give us the equiconsistency between them.)

Apart from Blackwell determinacy, I am interested in higher forcing absoluteness (Σ^2_n forcing absoluteness for a natural number n), descriptive set theory in $\mathcal{H}_{\omega_2}$, and the inner models constructed from first-order logics with generalized quantifiers (those obtained like L by replacing "first-order definable sets" by "definable sets by the logics").