RESEARCH STATEMENT

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My research is in Set-Theoretic/General Topology. More specifically, I have been working on h-homogeneity, CLP-compactness and their behaviour under products. A general fact that contributes to making those topics interesting is that clopen subsets of products need not be the union of clopen rectangles (see [2]).

A topological space X is *h-homogeneous* if all non-empty clopen subsets of X are homeomorphic (to X). The Cantor set, the rationals, the irrationals or any connected space are examples of h-homogeneous spaces. In [7], building on work of Terada (see [12]) and using Glicksberg's classical theorem on the Stone-Čech compactification of products, I obtained the following result.

Theorem 1. *Assume that X_i is zero-dimensional and h-homogeneous for every $i \in I$. Then $X = \prod_{i \in I} X_i$ is h-homogeneous.*

Furthermore, if X is pseudocompact, then the zero-dimensionality requirement can be dropped. (I don't know whether the zero-dimensionality requirement can be dropped in general.) Along the way, I showed that clopen subsets of pseudocompact products depend only on finitely many coordinates, thus generalizing a result of Broverman (see [1]). Also, I gave some partial answers to the following question from [12], which remains open.

Question 2 (Terada)**.** *Is X^ω h-homogeneous whenever X is zero-dimensional and first-countable?*

If one drops the 'h', then the answer is 'yes' by a remarkable theorem of Dow and Pearl (see [4]). Since h-homogeneity implies homogeneity for zero-dimensional first-countable spaces, a positive answer would give a strenghtening of their result. For other interesting papers on h-homogeneity, see [3], [5], [8], [9] or [13].

A topological space X is *CLP-compact* if every cover of X consisting of clopen sets has a finite subcover. For zero-dimensional spaces, CLP-compactness is the same as compactness. In [6], I obtained the following result, which answers a question of Steprāns and Šostak from [11]. The proof involves the construction of a special family of finite subsets of ω^*.

Theorem 3. *For every infinite cardinal κ, there exists a family $\{X_\xi : \xi \in \kappa\}$ such that $\prod_{\xi \in F} X_\xi$ is CLP-compact for every $F \in [\kappa]^{<\omega}$ while $\prod_{\xi \in \kappa} X_\xi$ is not.*

For a positive result on (finite) products of CLP-compact spaces, see [10].

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