Borel equivalence relations and Polish groups

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In many places in mathematics, important spaces are quotients of Polish spaces by Borel equivalence relations. In general, such quotients do not carry natural reasonable topologies or even Borel structures. One can, however, study them by exploring the equivalence relation used to form the quotient. Descriptive Set Theory provides tools for this exploration. In the tutorial, I will present basic such tools, and then I will concentrate on the roughest conjectural division of the class of Borel equivalence relations: each Borel equivalence relation is to be either Borel reducible to the orbit equivalence relation of a continuous action of a Polish group, or otherwise is to Borel reduce a complicated Borel equivalence relation called E_1. This dichotomy has been proved so far only in very restricted contexts. I will present the known results on this topic. This will lead us to the notion of Polishable group. I will present descriptive set theoretic structure of such groups and describe connections of this structure with other mathematical properties of groups.

All the material in my tutorial will come from the following papers.

1. A.S. Kechris, A. Louveau, *The structure of hypersmooth equivalence relations*, J. Amer. Math. Soc. 10 (1997), 215–242.

2. S. Solecki, *Polish group topologies*, in *Sets and Proofs*, pp. 339–364, London Math. Soc. Lecture Note Ser. 258, Cambridge Univ. Press, 1999.

3. I. Farah, S. Solecki, *Borel subgroups of Polish groups*, Adv. Math. 199 (2006), 499–541.

4. S. Solecki, *The coset equivalence relations and topologies on subgroups*, Amer. J. Math. 131 (2009), 571–605.