Prof. Dr. Matthias Lesch
Mathematisches Institut

Vorlesungsankündigung Wintersemester 2006/07

Atiyah-Singer Index Theory

The Atiyah-Singer Index Theory (according to Atiyah it is a theory and not just a theorem!) is one of the landmarks of Global Analysis. It contains as special cases the classical theorems of Gauß-Bonnet, Riemann-Roch and the celebrated Hirzebruch signature theorem. The theory reveals a deep connection between Topology, Geometry and Analysis of manifolds.

The above mentioned classical theorems all have in common that a certain topological invariant (e.g. Euler-Characteristic oder Signature) of a manifold equals the integral of a geometrically defined differential form. The Index Theory shows that the former is the Fredholm index of a natural elliptic differential operator while the latter is a characteristic differential form which can be extracted from the coefficients of the operator. The index theorem generalizes this concept in various directions: foremost it shows that the index of an arbitrary elliptic (pseudo)differential operator can be calculated in terms of topological invariants of its symbol. More sophisticated generalizations deal with families of operators or with operators which are invariant under a group or a Clifford action.

The goal of this two semester course is to provide an introduction to this fascinating theory. In the first semester we will discuss the so called embedding proof of the Index Theorem. Major subjects are

- K-theory
- Characteristic Classes
- Embedding proof of the Index Theorem
- Clifford algebras, Dirac operators and Spin geometry
- Applications in geometry and topology

Prerequisites Linear Algebra I-II, Analysis I-III and basic knowledge of differential topology. Basic knowledge of microlocal analysis (e.g. as provided by my previous course on analysis on manifolds) would be helpful but is not mandatory. I will give a short overview of the necessary facts. It is highly recommended to attend the Übungen.

Literature: I will mainly follow the book by Lawson and Michelsohn

- Berline, Getzler, Vergne: *Heat kernels and Dirac operators*, Springer 1992
- Booss, Bleecker: *Topology and analysis*, Springer 1985
- Gilkey: *Invariance theory, the heat equation, and the Atiyah-Singer index theorem*, Second edition, 1995
- Lawson, Michelsohn: *Spin geometry*, Princeton Univ. Press, 1989
- Roe, *Elliptic operator, topology, and asymptotic methods,* Second edition, Addison Wesley, 1998
- Rosenberg, *The Laplacian on a Riemannian manifold*, Cambridge Univ. Press, 1997

when and where: Di, Do 10-12, Seminarraum B.

Übungen: 2 hours, time TBA. Highly recommended.

Contact: Benjamin Himpel (Be 4/36 himpel@math.uni-bonn.de), Michael Bohn (Be 4/36 mbohn@math.uni-bonn.de), Matthias Lesch (Be6/26)