

Vorlesungsankündigung Sommersemester 2007 **Atiyah-Singer Index Theory II**

The goal of this two semester course is to provide an introduction to Atiyah-Singer Index Theory. After having discussed the classical embedding proof of the Index Theorem in the framework of K-Theory in the Winter term, the goal of the second part now is study the more recent *Heat Equation method*.

This method has its origin in work of Atiyah-Bott and McKean-Singer who discovered that the index of a differential operator can be expressed by the trace of the so-called *Heat operator*. Studying the asymptotic expansion of its integral kernel leads to an expression of the index of an elliptic differential operator as the integral over local expressions. For Dirac operators, these terms can be computed thus giving another proof of the cohomological version of the Index Theorem.

However, the importance of the Heat Equation method goes much beyond that. One major point is, for example, that it is very well-suited for Index Theory on manifolds with boundary. This particular application will be discussed after a short digression on boundary value problems for Dirac operators.

Major subjects are

- Clifford algebras, Dirac operators and Spin geometry
- The Heat Kernel and the McKean-Singer formula
- The Heat Expansion
- The local Index Theorem for Dirac operators
- Zeta- and Eta-Functions
- Spectral boundary conditions
- Index Theory on manifolds with boundary

Prerequisites: Linear Algebra I-II, Analysis I-III and basic knowledge of differential topology. Although this is a follow-up course to my course in the Winter term, many parts of the lecture will be self-contained. Some knowledge of differential geometry would be helpful but is not mandatory. I will give a short overview of the necessary facts. It is highly recommended to attend the Übungen.

Literature: For the first part of the semester I will mainly follow the book by Berline, Getzler and Vergne

- Berline, Getzler, Vergne: *Heat kernels and Dirac operators*, Springer 1992
- Gilkey: *Invariance theory, the heat equation, and the Atiyah-Singer index theorem*, Second edition, 1995
- Lawson, Michelsohn: *Spin geometry*, Princeton Univ. Press, 1989
- Roe, *Elliptic operator, topology, and asymptotic methods*, Second edition, Addison Wesley, 1998

When and where: Wed, Fri 10-12, Seminarraum D

Übungen: 2 hours, time TBA. Highly recommended.